

CHyPS: An MFEM-Based Material Response Solver for Hypersonic Thermal Protection Systems

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FEM@LLNL Seminar Series

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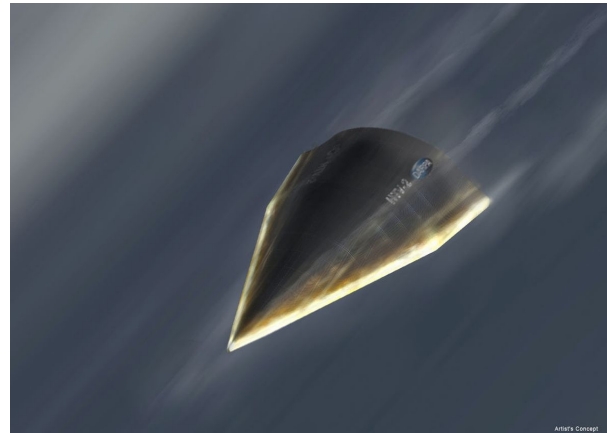
Challenges of Hypersonic Flight

Hypersonic flight leads to a wealth of technical challenges

- Speeds exceeding 4,000 miles per hour
- Surface temperatures exceeding that of the sun
- Communication blackout due to gas ionization
- Vehicle guidance and control
- ... and many more



X-51 Waverider (Mach 5)



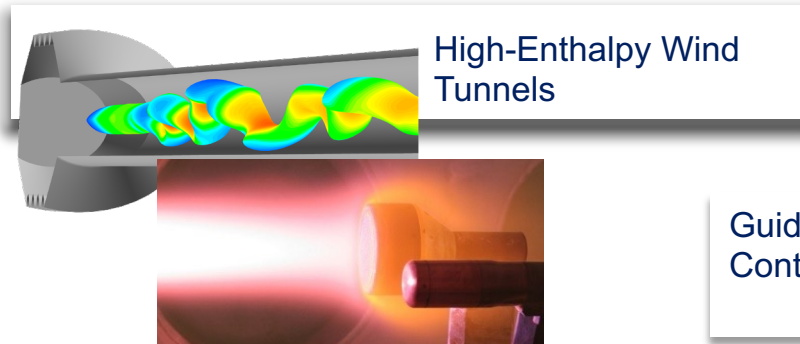
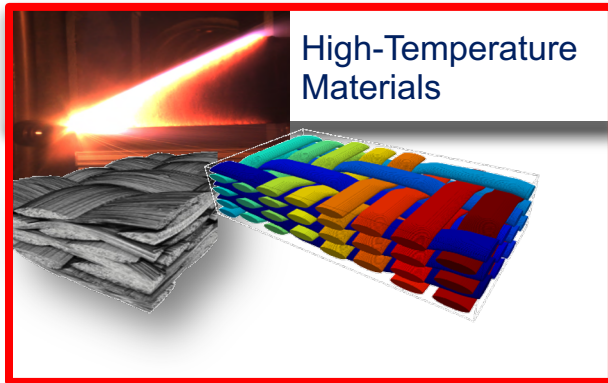
HTV-2 (Mach 20+)



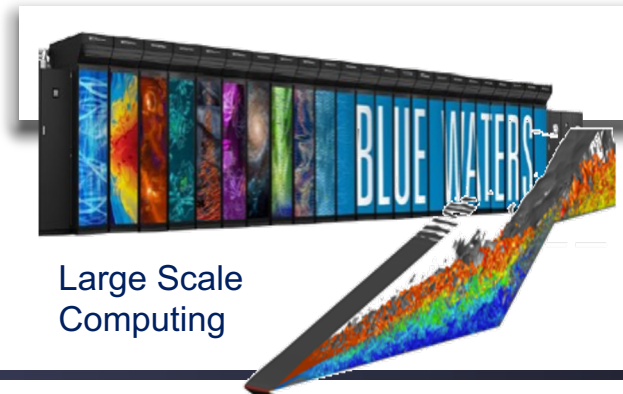
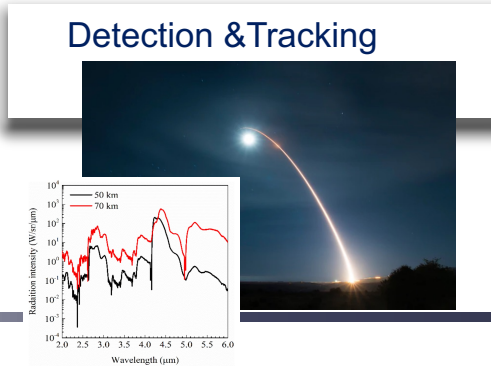
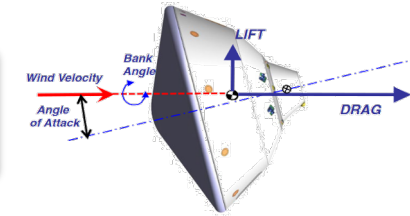
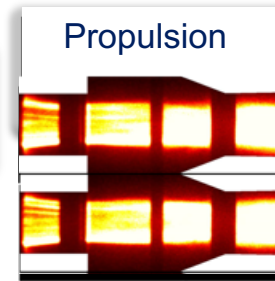
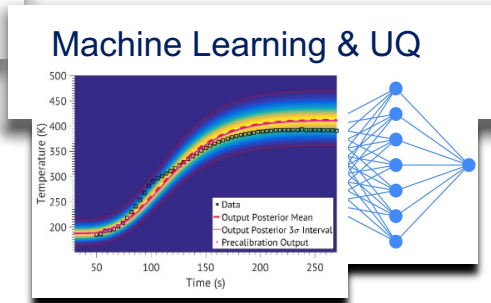
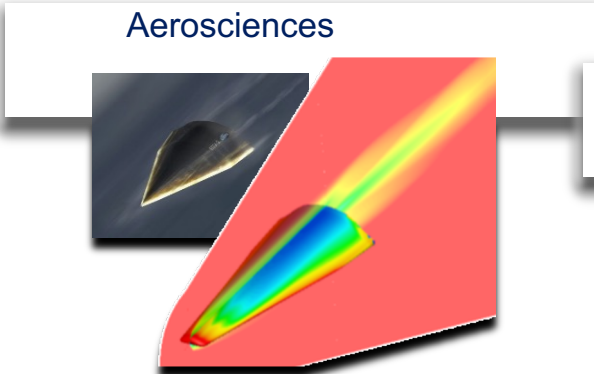
Stardust (Mach 43)



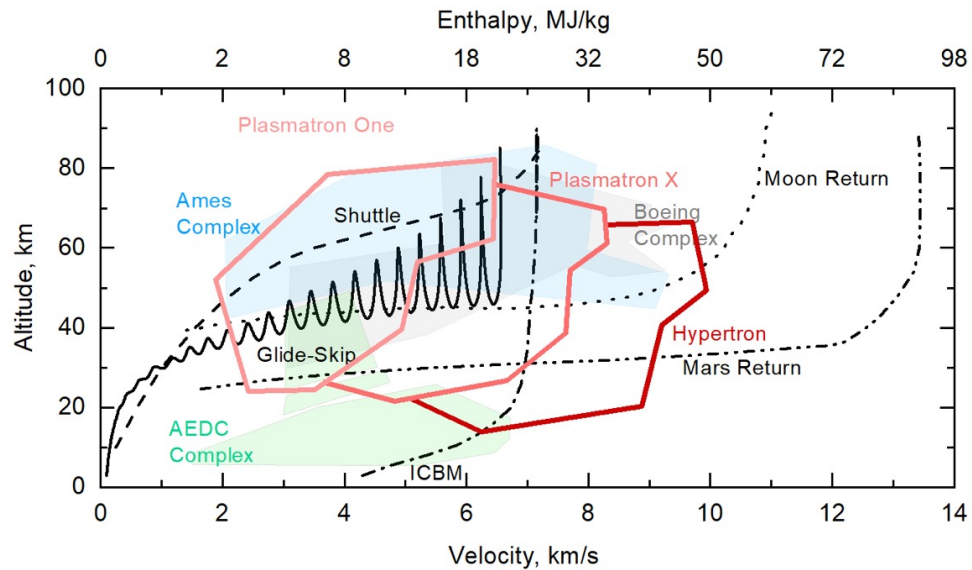
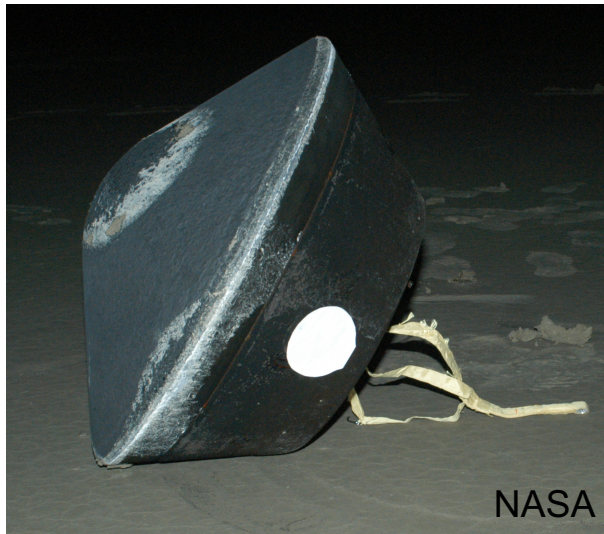
Center for Hypersonics and Entry-Systems Studies



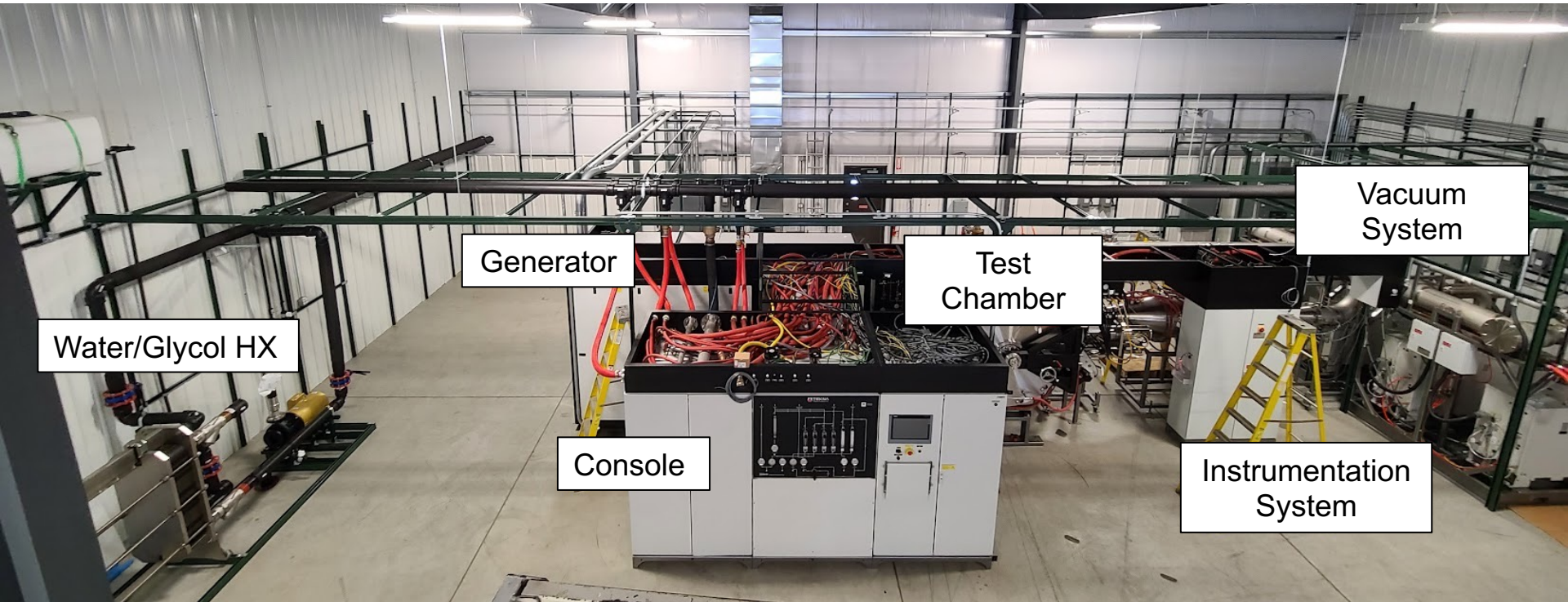
Guidance, Navigation & Control



Material Response during Hypersonic Flight

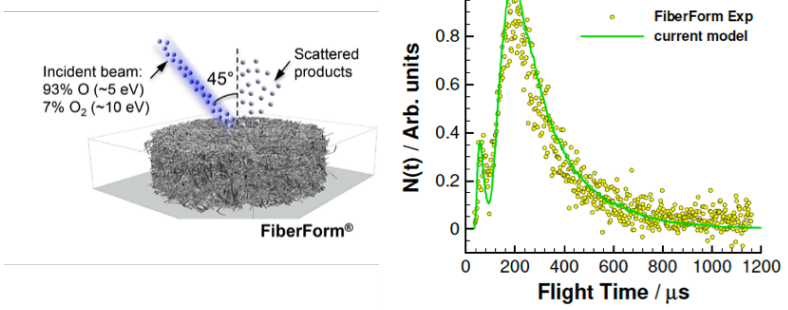
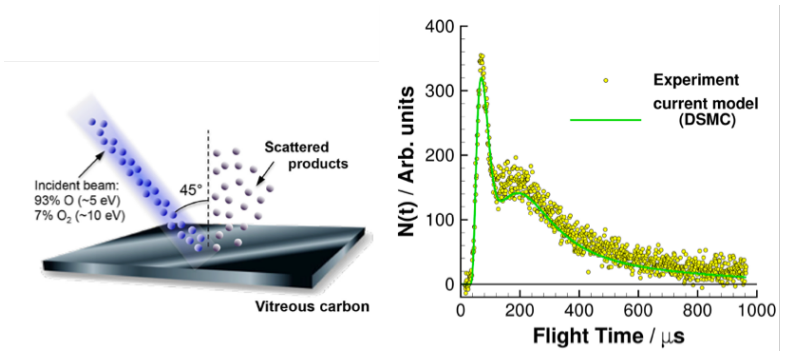


CHES Experimental Capabilities

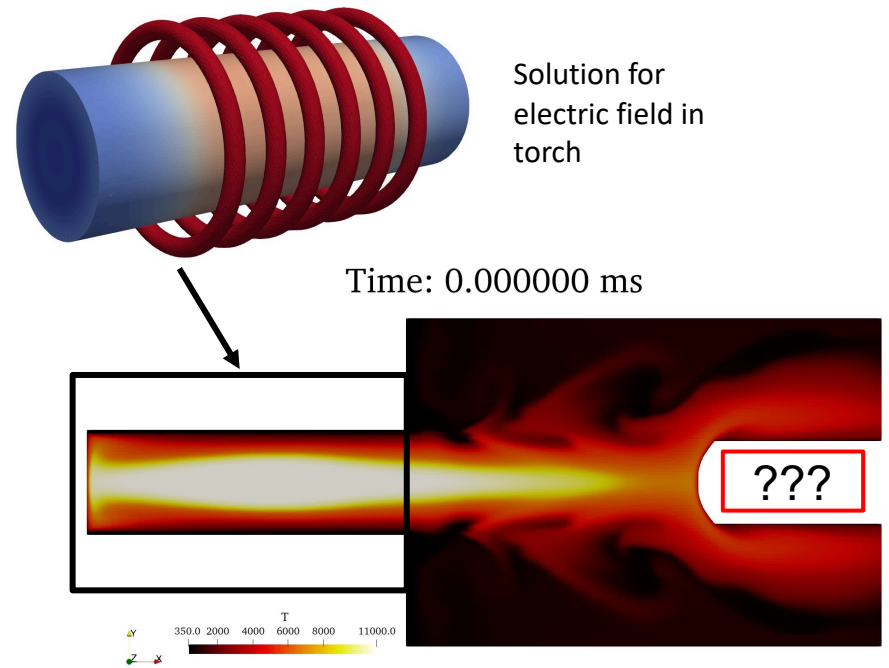


Newly completed Plasmatron-X

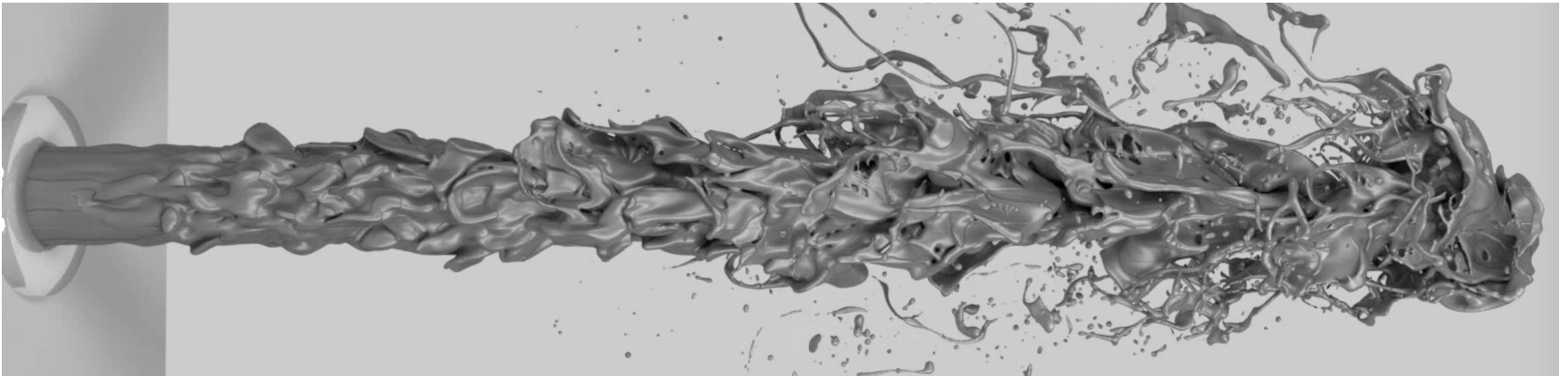
CHES Computational Capabilities



DOE SPARTA DSMC

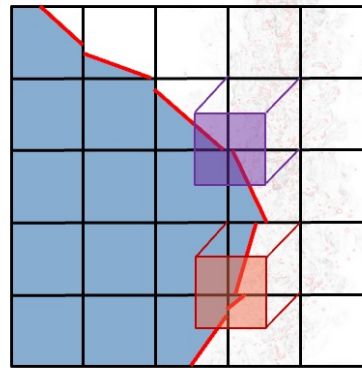
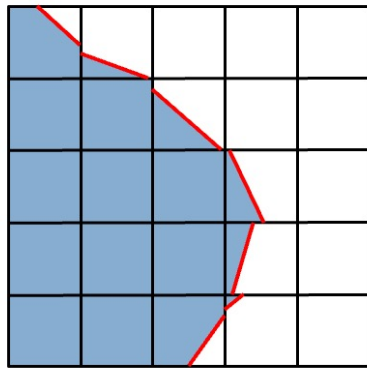
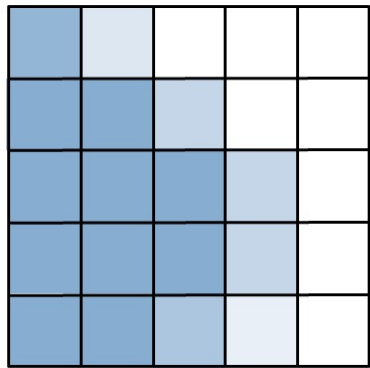


My Background is not in FEM...



Reconstruction

Advection

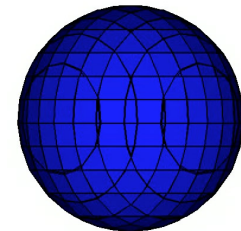


Volume Fraction Field (α)

Model interface as plane (PLIC)

Unsplit Geometric Advection

$$\hat{n} \cdot \vec{x} - d = 0$$



Requirements for a New Material Response Tool

Goal: A state of the art material response solver that can perform full-vehicle simulations and readily couple to external solvers to add additional physics

Tool features required for full-vehicle simulation

- Scalability to large problems and core counts
- Faithful representation of vehicle geometry features
- Robustness to low quality meshes
- Correct level of abstraction and flexibility for physical model

Targeting cases where coupled response is important

- Material response of control surfaces and thin-ablators
- Boundary layer stability and transition on ablating surfaces
- Development of small-dimension models for guidance and control



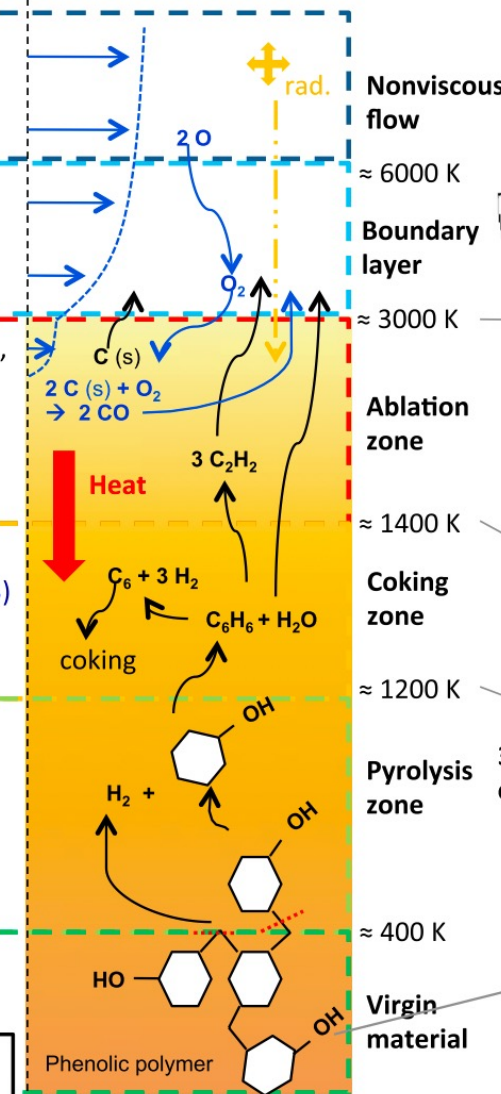
Macroscopic phenomenology

Macroscopic illustration

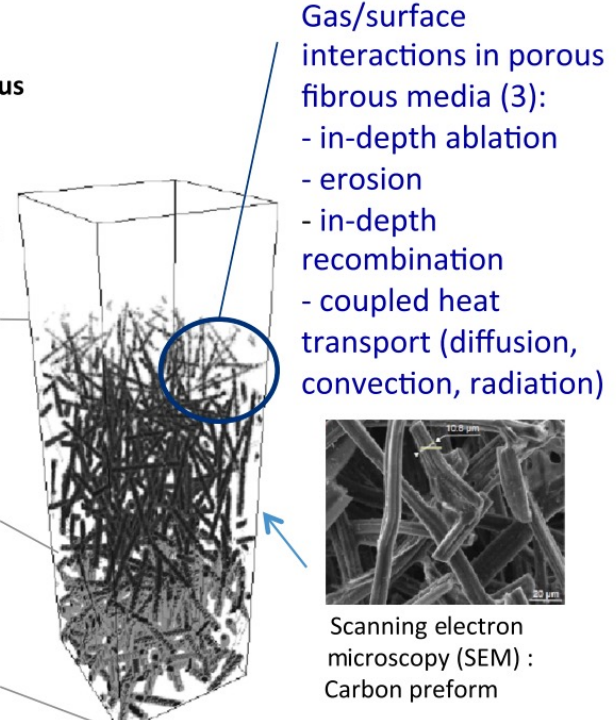
Microscopic illustration

- Hypersonic flow**
- Long distance effects
 - Radiation
 - Material-flow coupling
 - Boundary layer transfers (heat and mass)
 - Recombination/catalicity
 - Ablation (oxidation, sublimation, spallation)
- Material response**
- Interface phenomena: Heat and mass balance (1), Subsurface phenomena
 - In-depth ablation (3)
 - Penetration of radiation (3)
 - Gas flow entering into the material (3)
 - Conduction heat transfer (1)
 - Radiation heat transfer (empirical: 1, modeled: 3)
 - Finite rate chemistry of the pyrolysis gases (3)
 - Coking (3)
 - Multicomponent diffusion (3)
 - Convective transport (Darcy: 2, Klinkenberg: 3)
 - Charring process (evolution of the density: 1, porosity: 2, tortuosity: 3, permeability: 2, effective conductivity: 1, effective surface area: 3)
 - Phenolic-decomposition product (3)
 - Phenolic-decomposition rate (1)

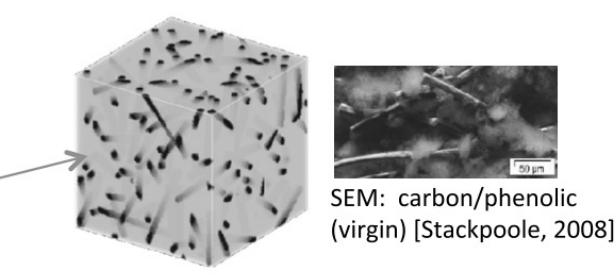
1 : in all material response models (type 1)
 2 : in some material response models (type 2)
 3 : in analysis material response models (type 3)



Chemistry mechanisms (simplified illustration)

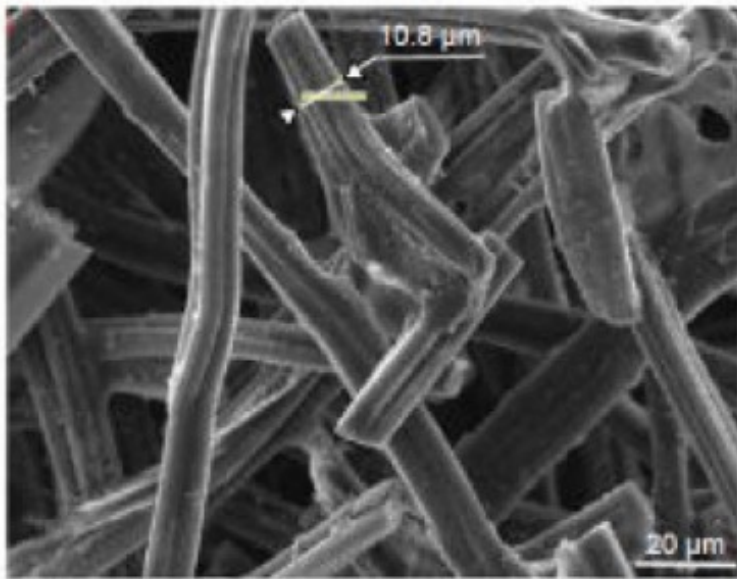
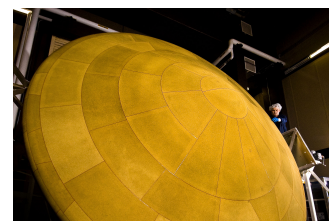


3-D simulation of the ablation of a carbon/phenolic composite [Lachaud [6]]

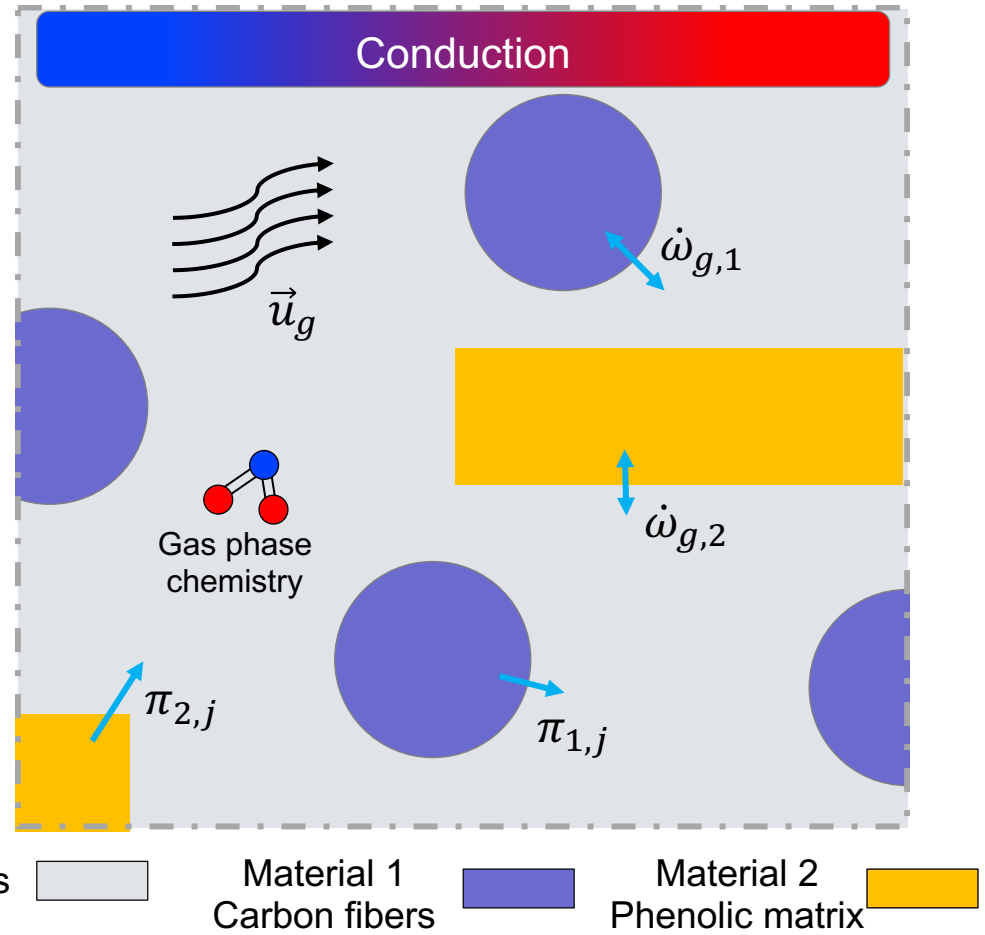


3-D reconstitution of a carbon/phenolic composite

Governing Physics



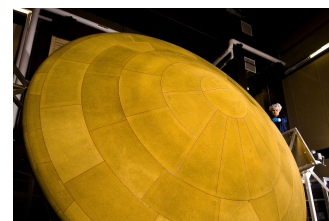
SEM of carbon preform prior to phenolic impregnation [1]



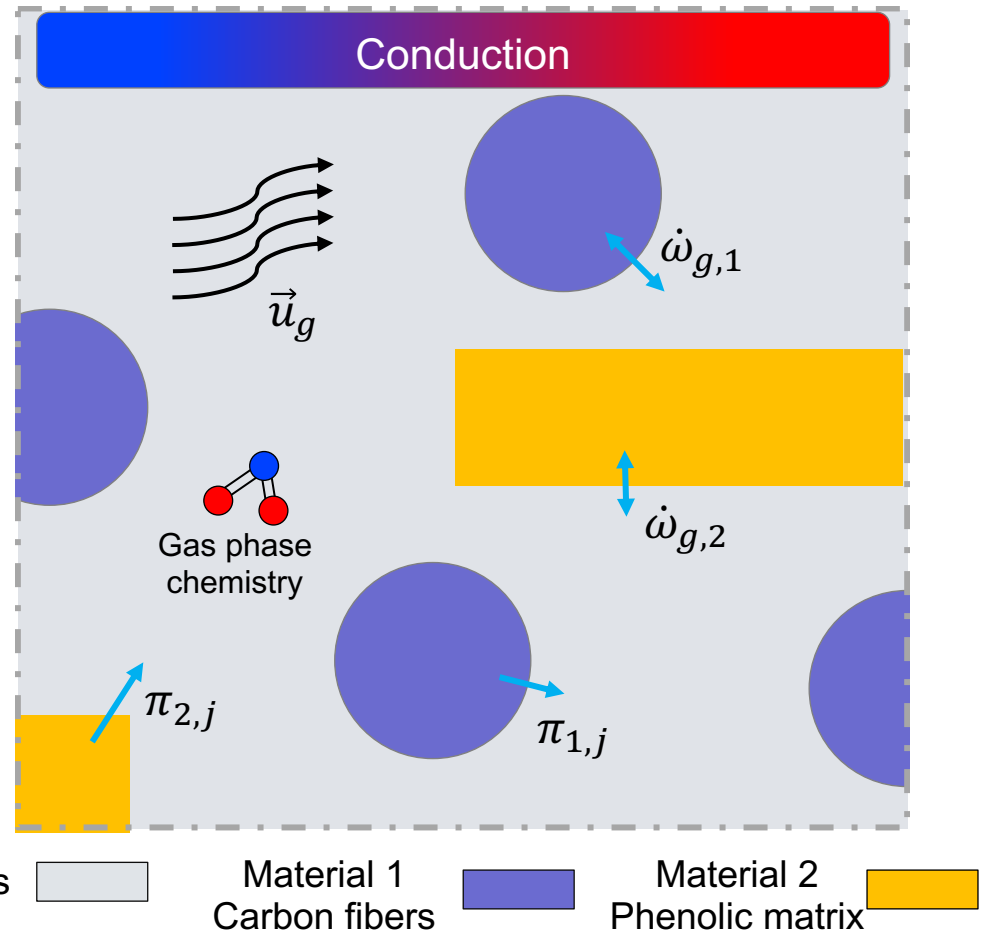
[1] Lachaud et al. IJHMT. 2015



Governing Physics



<p>Gas properties</p> <p>Volume fraction ϵ_g</p> <p>Pressure P_g</p> <p>Enthalpy h_g</p> <p>Density ρ_g</p> <p>Viscosity μ_g</p>
<p>Material properties ($i \in [1, N_p]$)</p> <p>Volume fraction ϵ_i</p> <p>Density ρ_i</p> <p>Specific heat $c_{p,i}$</p> <p>Enthalpy h_i</p> <p>Pyrolysis specie production $\pi_{i,k}$</p>
<p>Bulk properties</p> <p>Temperature T (thermal equilibrium)</p> <p>Conductivity κ</p> <p>Permeability K</p>



[1] Lachaud et al. IJHMT. 2015



Volume-Averaging over a Representative Elementary Volume

Smallest volume that is representative of the bulk properties of the whole

Phase indicator functions:

$$H(\vec{x}) = \begin{cases} 1, & \vec{x} \in \text{phase} \\ 0, & \vec{x} \notin \text{phase} \end{cases}$$

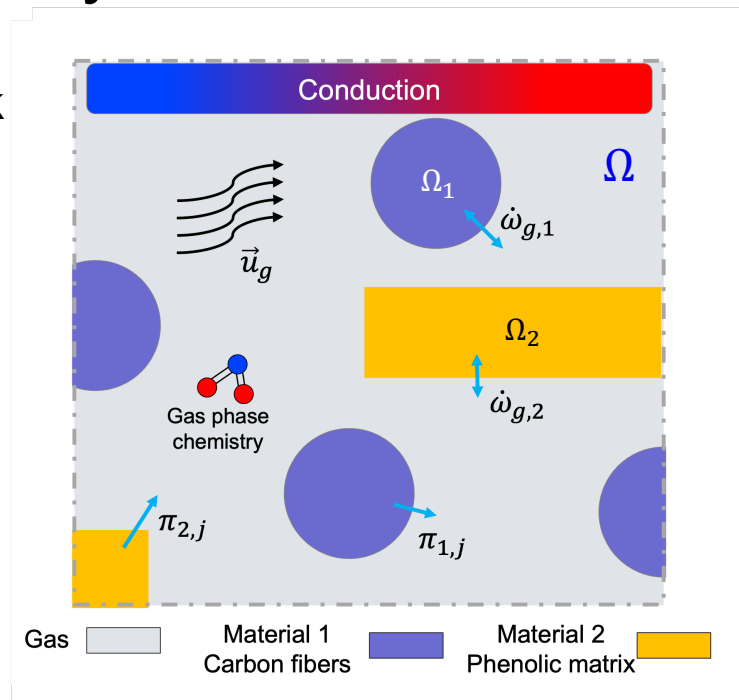
For a fiber/matrix composite, three total phases

Volume fraction:

$$\epsilon_i = \frac{\int_{\Omega} H_i(\vec{x}) d\vec{x}}{\int_{\Omega} d\vec{x}} \quad \text{or} \quad \epsilon_i = \frac{\int_{\Omega_i} d\vec{x}}{\int_{\Omega} d\vec{x}}$$

Average solid density:

$$\rho_s = \frac{\sum_{i \in N_p} \int_{\Omega_i} \rho_i d\vec{x}}{\int_{\Omega} d\vec{x}} = \sum_{i \in N_p} \epsilon_i \rho_i$$



Governing Equations

Solid mass conservation:

$$\frac{\partial \epsilon_i \rho_i}{\partial t} = - \sum_{k \in [1, N_{sp}]} \pi_{i,k}$$

Gas mass conservation equation with ideal gas law and Darcy's law assumption

Gas mass/momentum conservation:

$$\frac{\partial}{\partial t} \left(\frac{\epsilon_g M P_g}{RT} \right) - \nabla \cdot \left(\frac{P_g M}{RT \mu} K \nabla P_g \right) = \sum_{\substack{i \in [1, N_p], \\ k \in [1, N_{sp}]} \pi_{i,k}, \quad \vec{u}_g = - \frac{K}{\epsilon_g \mu} \nabla P_g$$

Energy conservation:

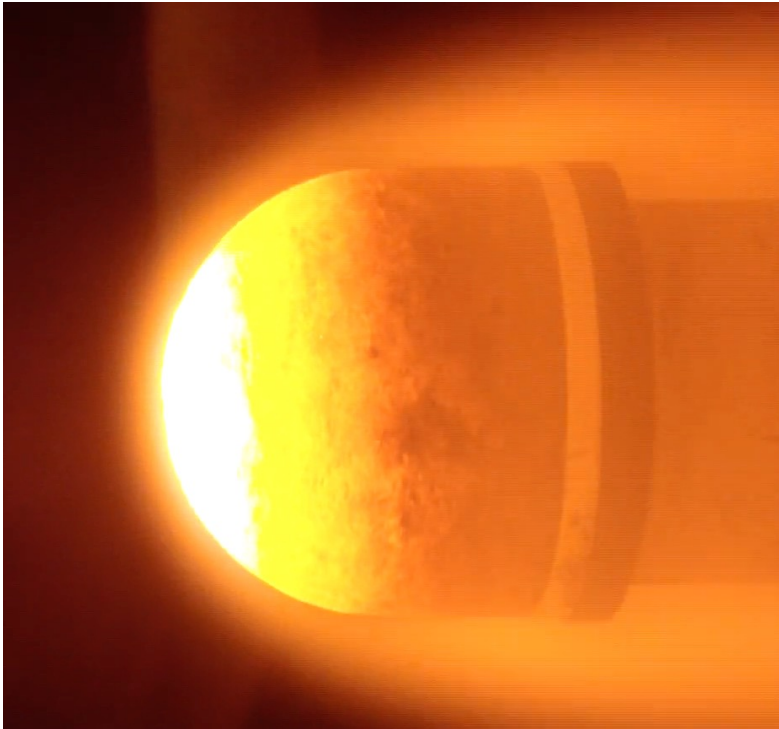
$$\left(\sum_{i \in [1, N_p]} \epsilon_i \rho_i c_{p,i} \right) \frac{\partial T}{\partial t} - \nabla \cdot \kappa \nabla T = - \sum_{i \in [1, N_p]} h_i \frac{\partial \epsilon_i \rho_i}{\partial t} - \frac{\partial (\epsilon_g \rho_g h_g - \epsilon_g P_g)}{\partial t} - \nabla \cdot (\epsilon_g \rho_g h_g \vec{u}_g)$$

Pyrolysis chemistry:

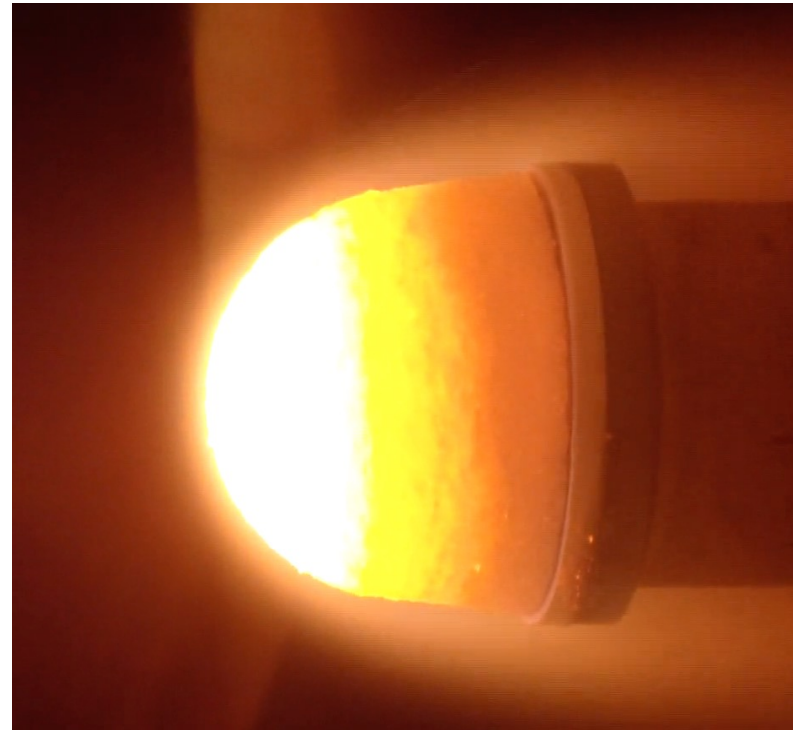
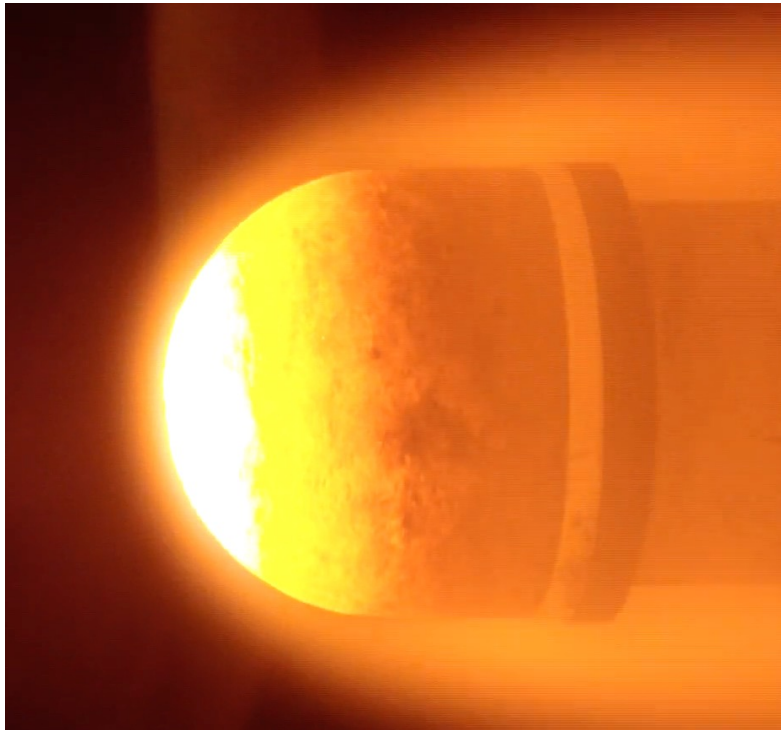
$$\frac{\partial X_{i,j}}{\partial t} = (1 - X_{i,j})^{m_{i,j}} T^{n_{i,j}} A_{i,j} \exp \left(- \frac{E_{i,j}}{RT} \right), \quad \pi_{i,k} = \sum_{j \in [1, P_i]} \zeta_{i,j,k} \epsilon_{i,0} \rho_{i,0} F_{i,j} \frac{\partial X_{i,j}}{\partial t}$$



Ablative Boundary Condition

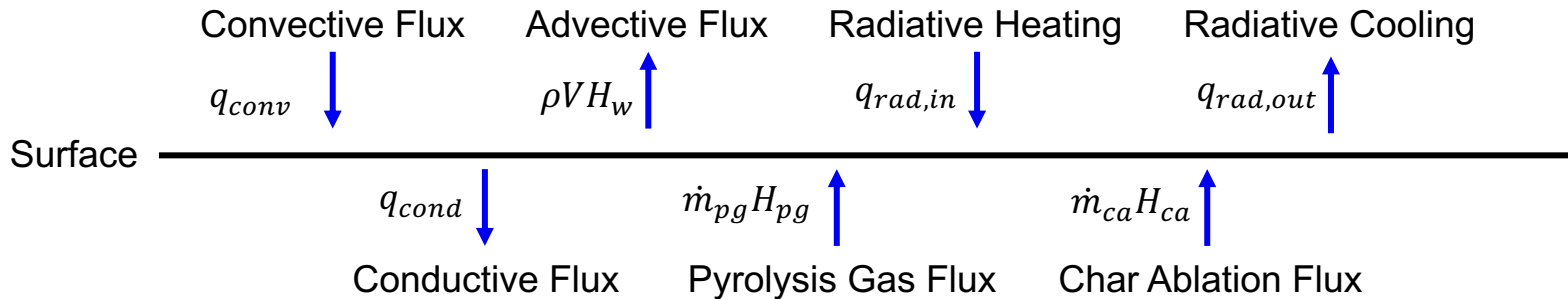


Ablative Boundary Condition



Ablative Boundary Condition

Ablative surface boundary condition from local equilibrium assumption



Provides Neumann boundary condition (heat flux) for energy equation

$$q_{cond} = q_{conv} - \rho V H_w + q_{rad,in} - q_{rad,out} + \dot{m}_{pg} H_{pg} + \dot{m}_{ca} H_{ca}$$

B' table is used to compute \dot{m}_{ca} and H_w as a function of $f(P, T, \dot{m}_{pg})$

\dot{m}_{ca} directly controls the ablative surface recession



Governing Equations - ALE

An Arbitrary Lagrangian Eulerian (ALE) used to allow moving mesh

Employ the conservative direct ALE method of Ivančić et al. 2019¹

Consider the heat equation $\frac{\partial u}{\partial t} \Big|_{\hat{x}} - \alpha \nabla^2 u - \vec{w} \cdot \nabla u = f$ in weak form with backward Euler time-integration

$$\int_{\hat{\Omega}} \hat{\psi} \hat{u}_{n+1} \hat{J}_{n+1} d\hat{x} - \int_{\hat{\Omega}} \hat{\psi} \hat{u}_n \hat{J}_n d\hat{x} + d_{n,n+1}(\hat{u}_{n+1}, \hat{\psi}) - b_{n,n+1}(\hat{u}_{n+1}, \hat{\psi}) - \mathcal{M}_{n,n+1}(\hat{u}_{n+1}, \hat{\psi}) = 0$$

$$d_{n,n+1}(\hat{u}_k, \hat{\psi}) = \Delta t \int_{\hat{\Omega}} \alpha \frac{1}{\hat{J}_{n,n+1}(\Delta t)} \hat{F}_{n,n+1}(\Delta t) \hat{F}_{n,n+1}^T(\Delta t) \hat{\nabla} \hat{\psi} \cdot \hat{\nabla} \hat{u}_k d\hat{x}$$

$$b_{n,n+1}(\hat{f}_k, \hat{\psi}) = \Delta t \int_{\hat{\Omega}} \hat{\psi} \hat{f}_k \hat{J}_{n,n+1}(\Delta t) d\hat{x}$$

$$\mathcal{M}_{n,n+1}(\hat{u}_k, \hat{\psi}) = \int_{\hat{\Omega}} \hat{\psi} \left[\int_0^{\Delta t} \hat{F}_{n,n+1}(t) \hat{w}_{n,n+1}(t) dt \right] \hat{\nabla} \hat{u}_k d\hat{x} + \int_{\hat{\Omega}} \hat{\psi} \hat{u}_k \hat{\nabla} \cdot \left[\int_0^{\Delta t} \hat{F}_{n,n+1}(t) \hat{w}_{n,n+1}(t) dt \right] d\hat{x}$$



Multi-Stage BilinearForm

Created a Multi-Stage BilinearForm to implement the direct ALE method

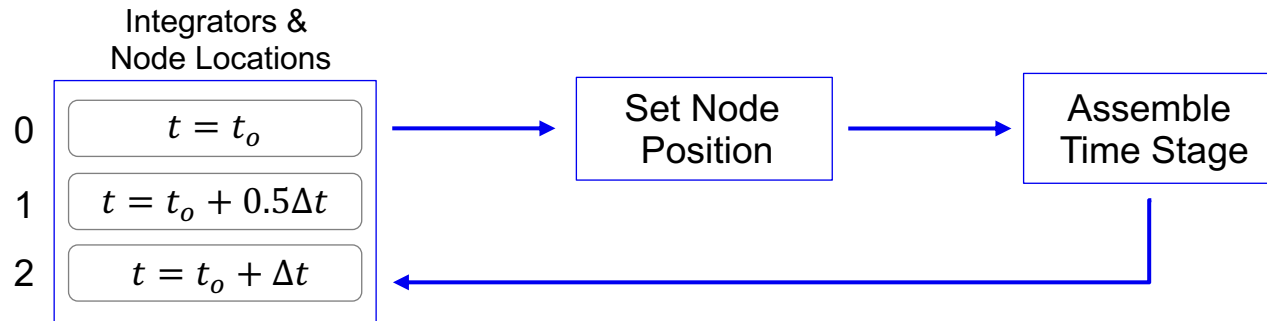
- Assumes integrating over $t \in [t_o, t_o + \Delta t]$ and known nodal displacement

Integrators are added to specific time-stages of the BilinearForm

- Added independently for each time stage
- Can include both domain and boundary integrators

Each time-stage has an associated GridFunction denoting nodal locations

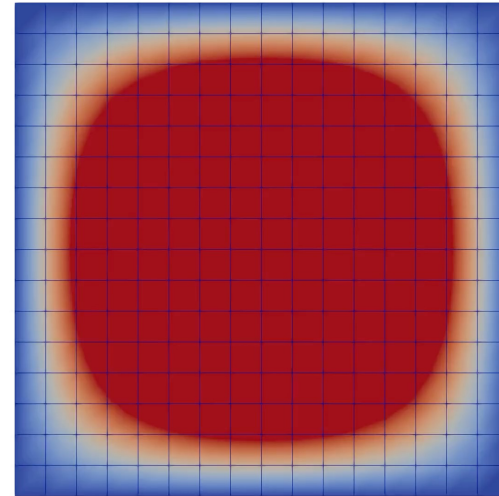
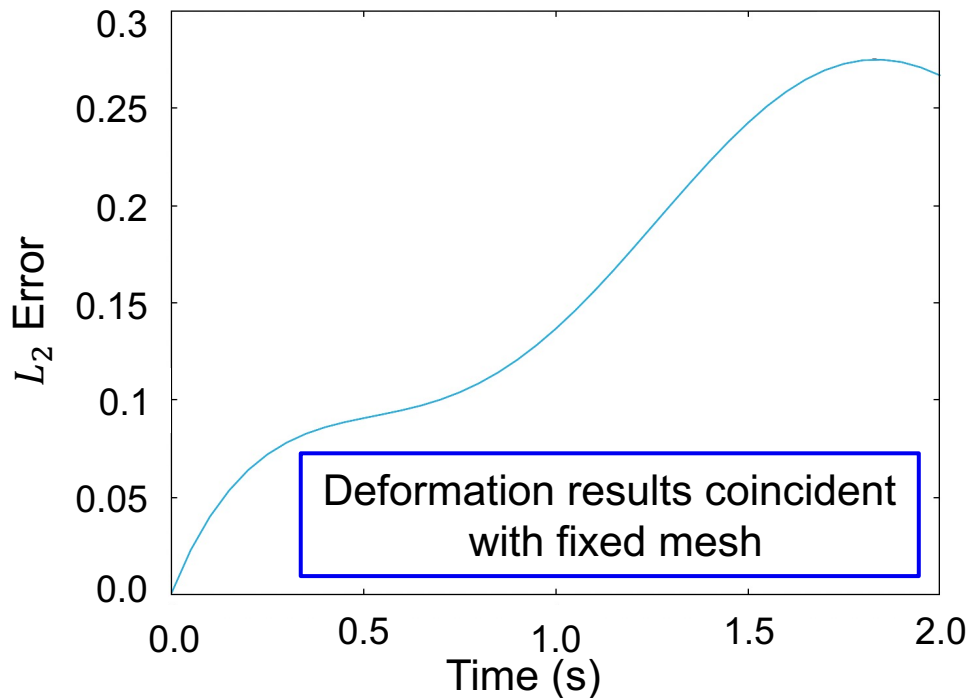
- Nodes set on Mesh before building of each time-stage
- References to nodes stored, allowing external update of locations



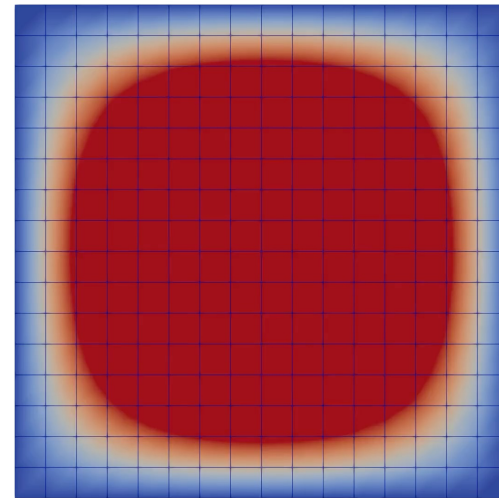
Governing Equations - ALE

$$\begin{aligned}\partial_t u - 0.1\Delta u &= f \text{ in } \Omega(t) \times (0, T), \\ u &= 0 \text{ on } \partial\Omega(t) \times (0, T), \\ u(0) &= 16x(1-x)y(1-y) \text{ in } \Omega(0)\end{aligned}$$

$$u(x, t) = 16 \left(1 + \frac{1}{2} \sin(\pi t) \right) x(1-x)y(1-y)$$



Deformation
A



Deformation
B

Implementation

Implemented in the Coupled Hypersonic Protection System (CHyPS) Simulator

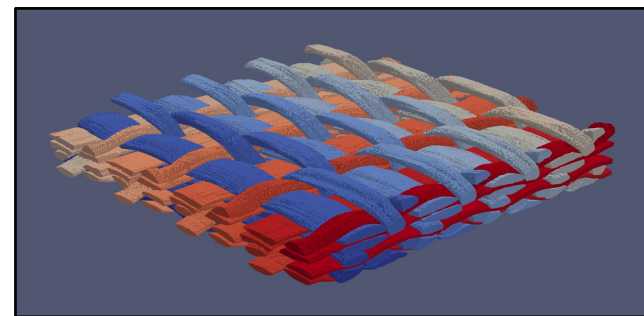
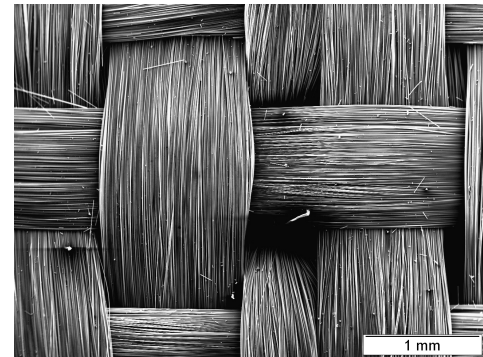
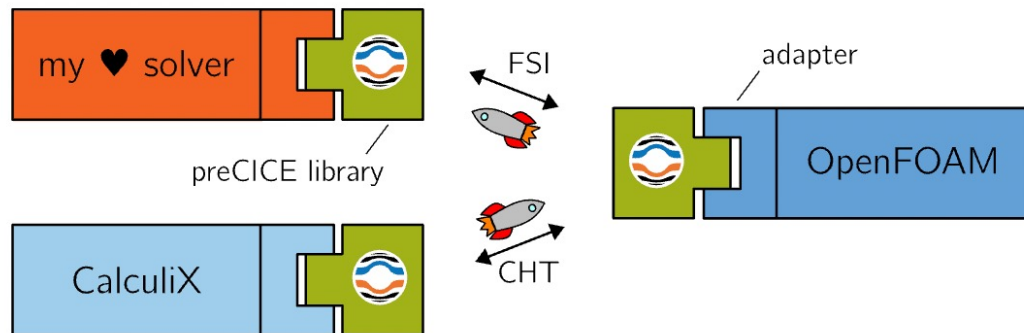
Discontinuous Galerkin spatial discretization via MFEM library [1]

Crank-Nicolson, Forward Euler, and Backward Euler time integration available

- Custom time-integration currently used, would like to return to MFEM ODE solvers

Tensor properties for conductivity and permeability

Coupling to external solvers using the preCICE library



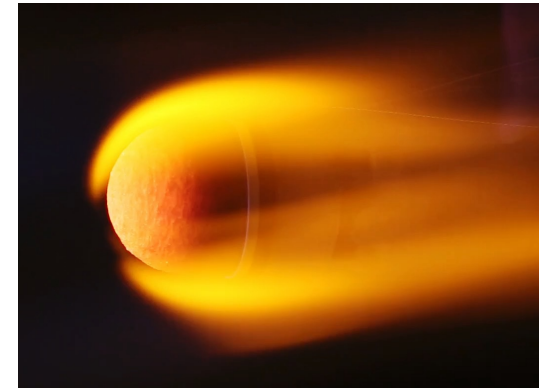
Ablation Test Case Series

Series of test cases developed over the last decade for code comparison

- Makes available open-results for a field dominated by national defense
- Feature Theoretical Ablative Composite for Open Testing (TACOT)
- Provide gradual increase in physical and computational complexity

Ablation Test Case 2

- Mimics the 1D heating of a sample in an arc jet facility
- Will compare against the Porous-Material Analysis Toolbox Based on OpenFOAM (PATO)



← Ablative BC, Temperature computed via energy balance

Heat for 60 s
Radiative cooling for 60 s

Material: TACOT
Length: 50 mm

Insulated
Impermeable

x



Ablation Test Case 2.3

Ablative BC, Temperature computed via energy balance

Heat for 60 s
Radiative cooling for 60 s

Material: TACOT
Length: 50 mm

Insulated
Impermeable

Temperature



320 K

3000 K



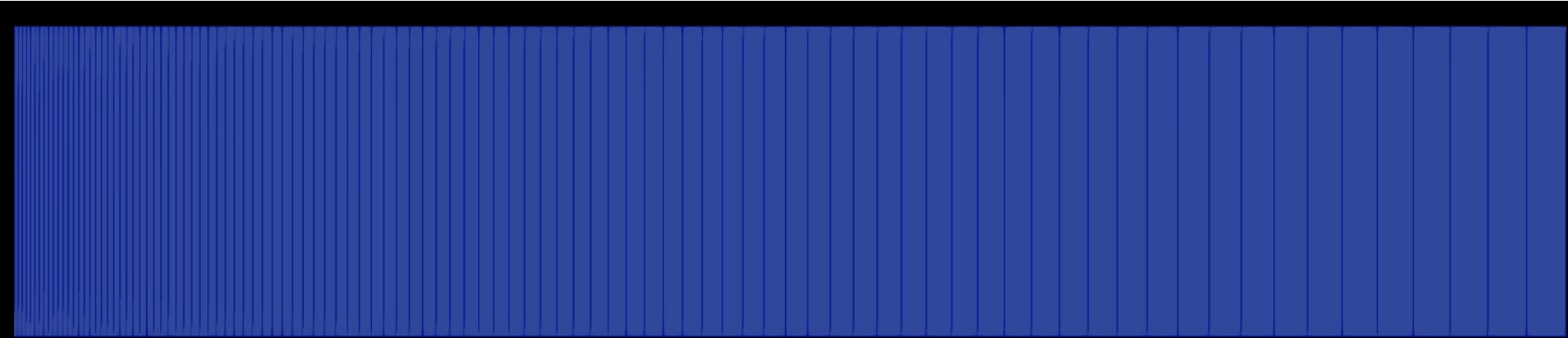
Ablation Test Case 2.3

← Ablative BC, Temperature computed via energy balance

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Radiative cooling for 60 s

Material: TACOT
Length: 50 mm

Insulated
Impermeable



Temperature



320 K

3000 K



Ablation Test Case 2.3

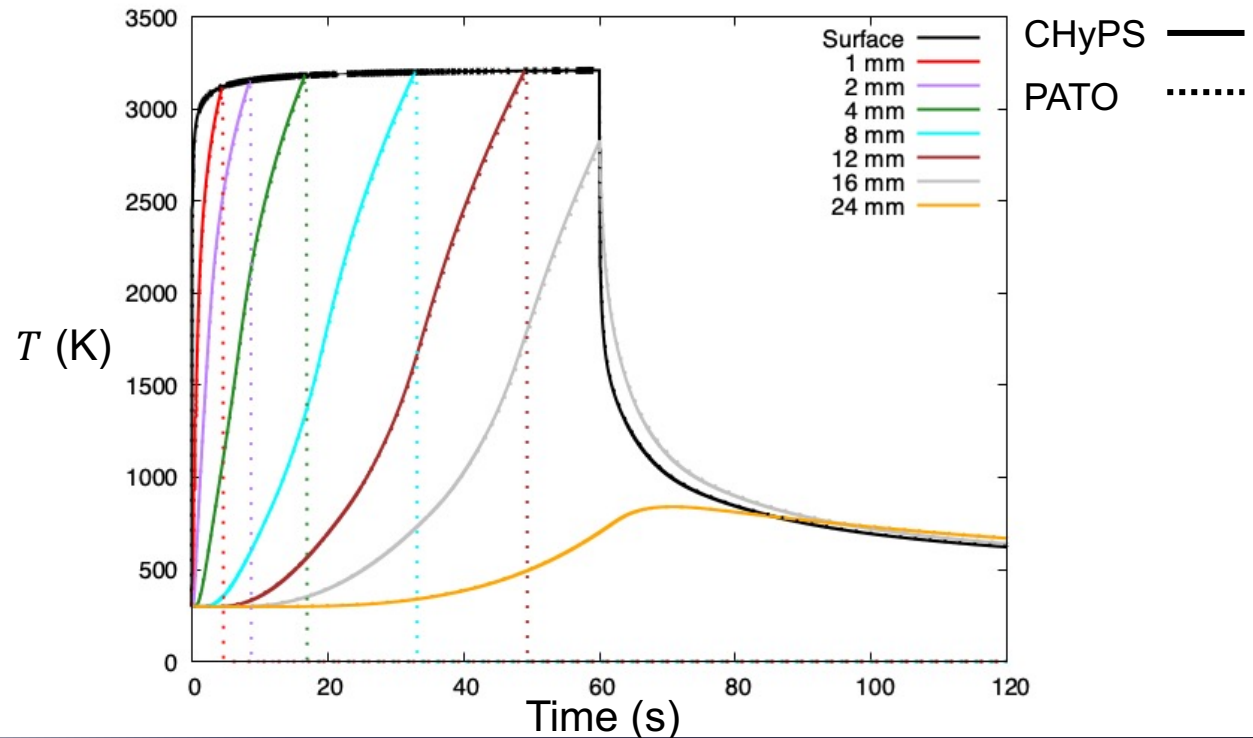
Ablative BC, Temperature computed via energy balance

Heat for 60 s
Radiative cooling for 60 s

Material: TACOT
Length: 50 mm

Insulated
Impermeable

Probe	x (mm)
Surface	0.0
1	1.0
2	2.0
3	4.0
4	8.0
5	12.0
6	16.0
7	24.0



Ablation Test Case 2.3

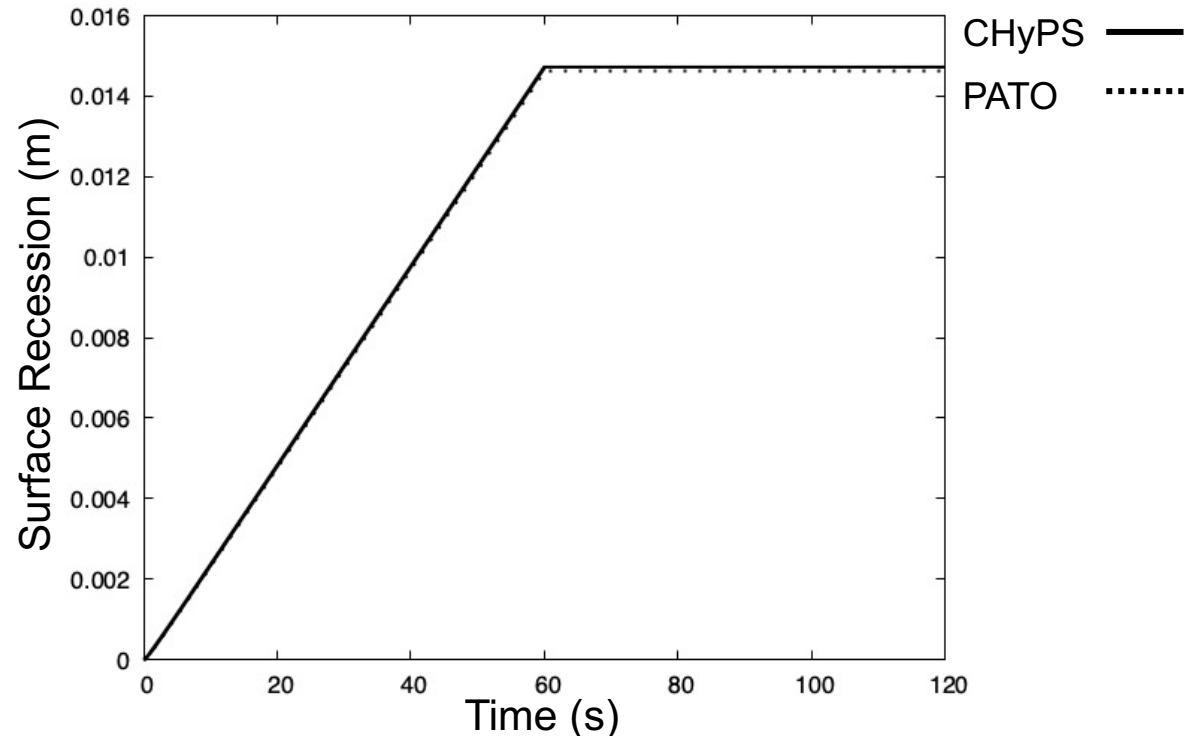
Ablative BC, Temperature computed via energy balance

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Impermeable

Probe	x (mm)
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6	16.0
7	24.0



Ablation Test Case 2.3

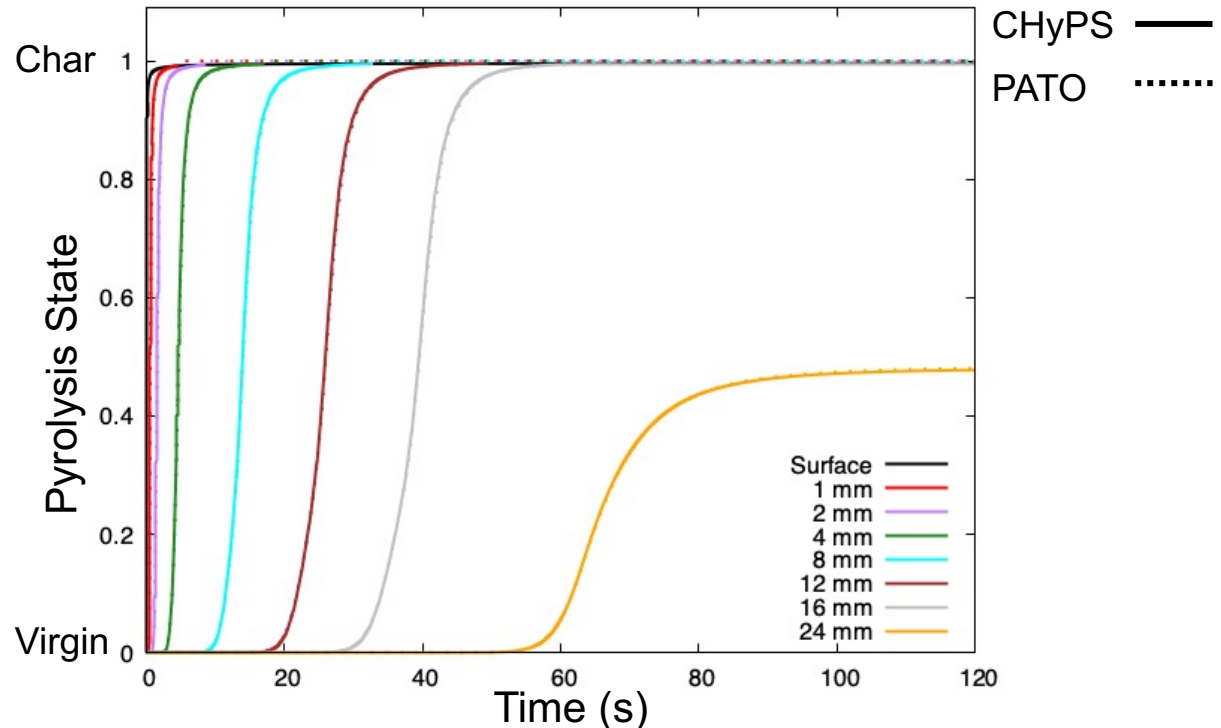
Ablative BC, Temperature computed via energy balance

Heat for 60 s
Radiative cooling for 60 s

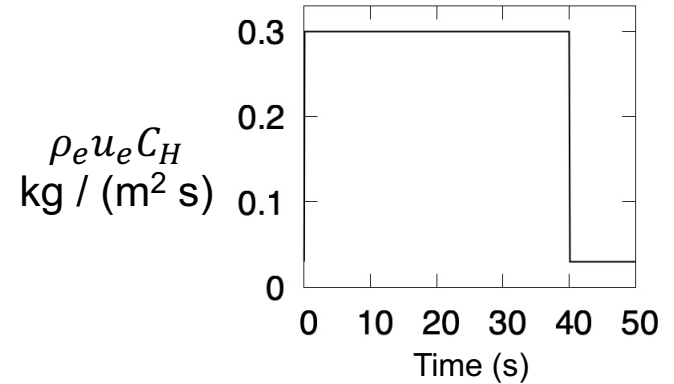
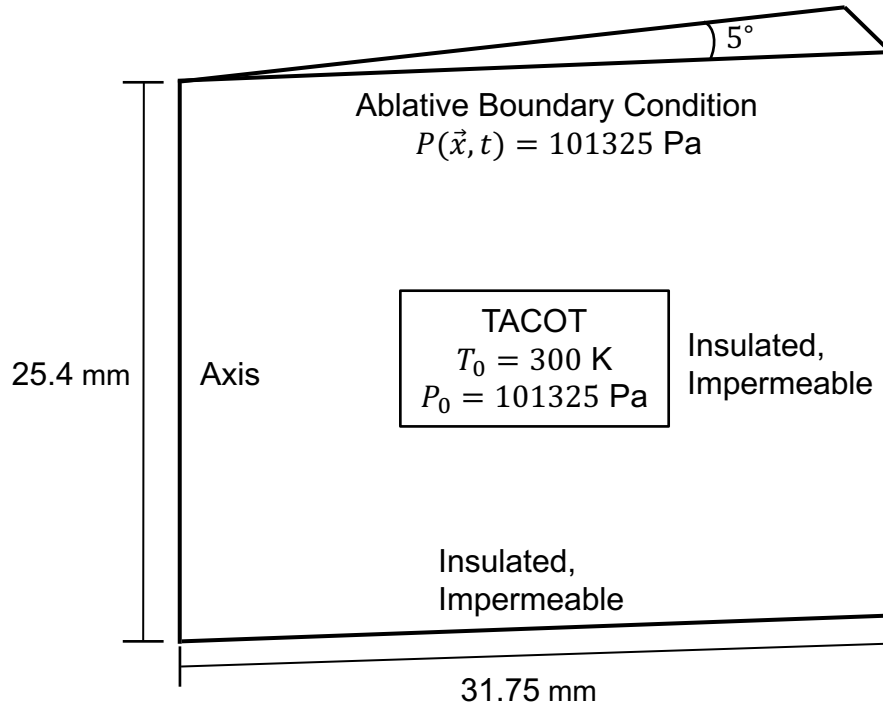
Material: TACOT
Length: 50 mm

Insulated
Impermeable

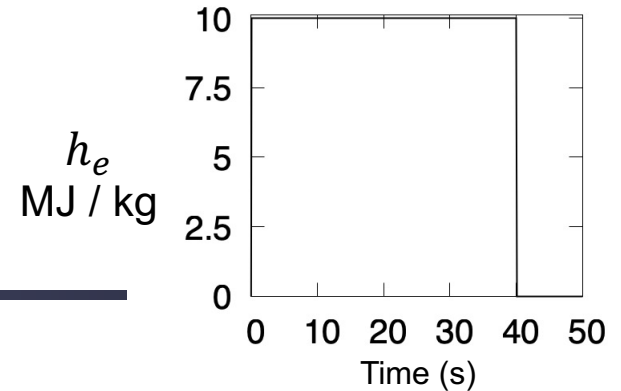
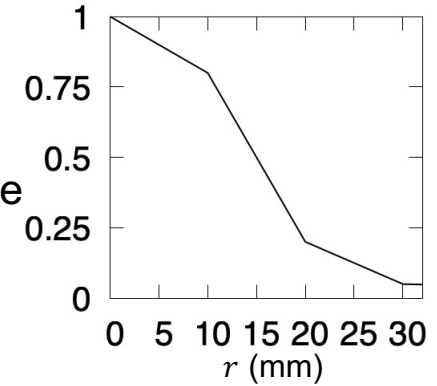
Probe	x (mm)
Surface	0.0
1	1.0
2	2.0
3	4.0
4	8.0
5	12.0
6	16.0
7	24.0



Axisymmetric Mini Arc Jet Case



Mapping of stagnation value



CHyPS

20 elements along radius
50 elements along height
DG, $p = 1$ polynomials

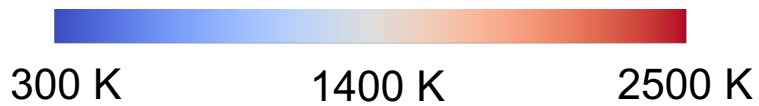
PATO

40 elements along radius
100 elements along height
Finite-volume method



Axisymmetric Mini Arc Jet Case

Temperature



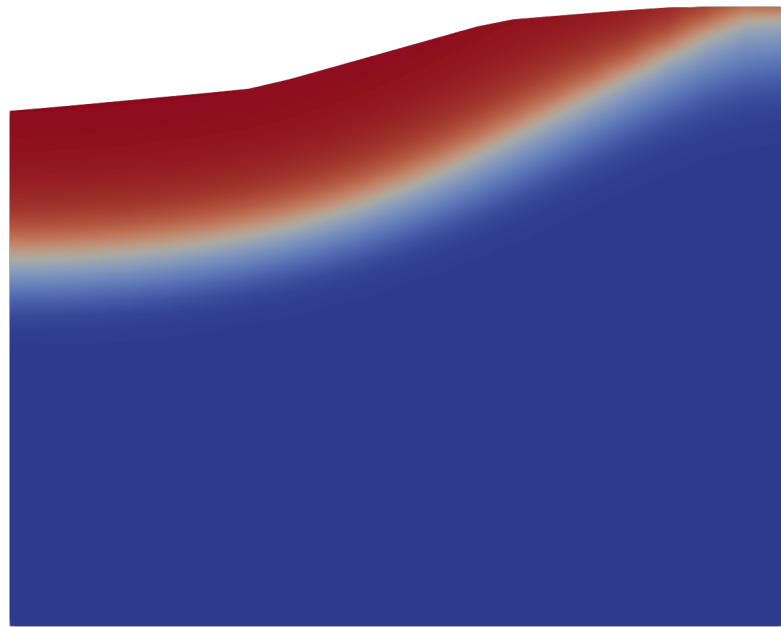
Gas Pressure



Axisymmetric Mini Arc Jet Case

$t = 20 \text{ s}$

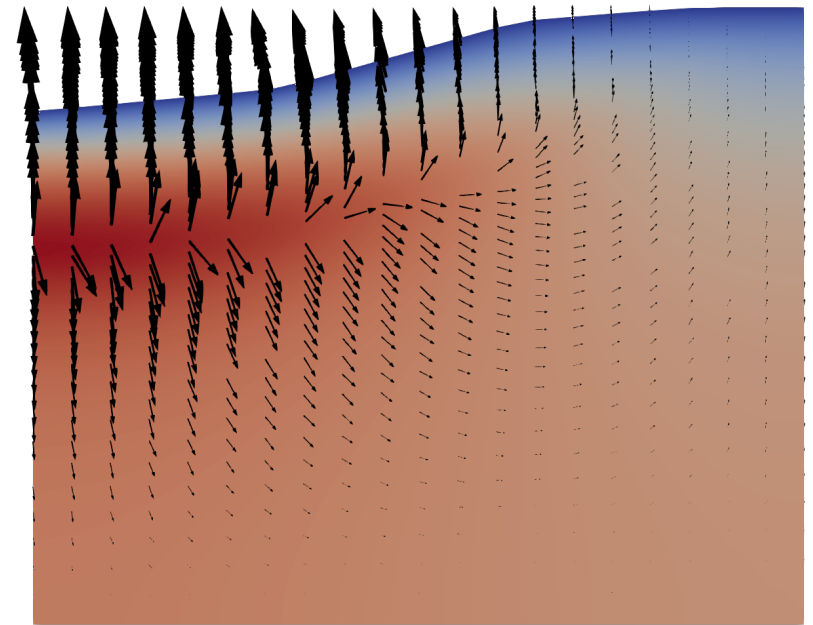
Pyrolysis State



0
Virgin

1
Char

Gas Pressure and Velocity

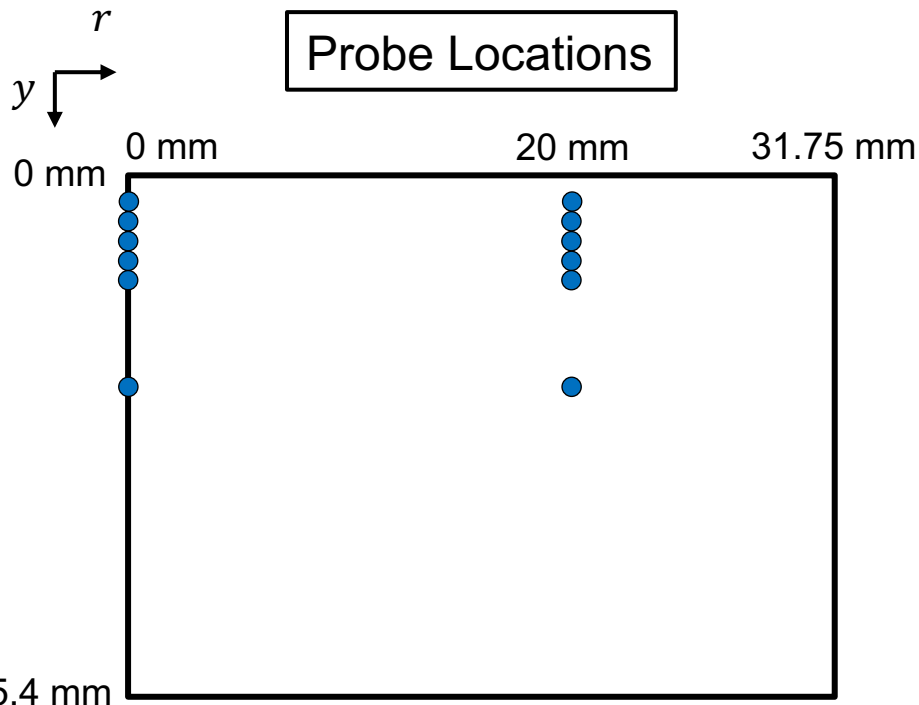


101325 Pa

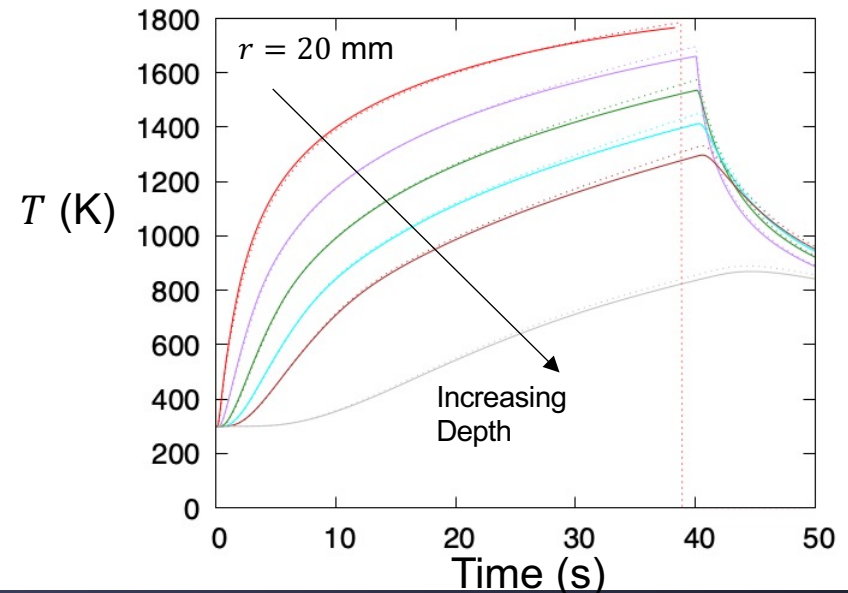
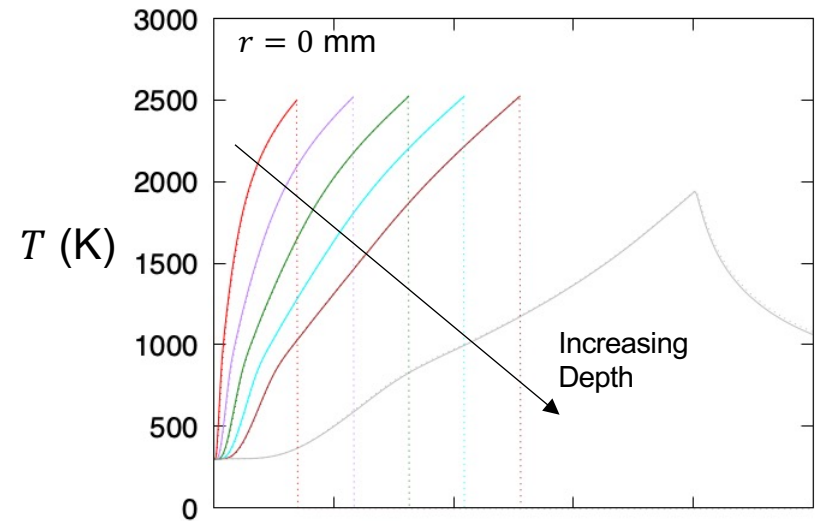
102350 Pa

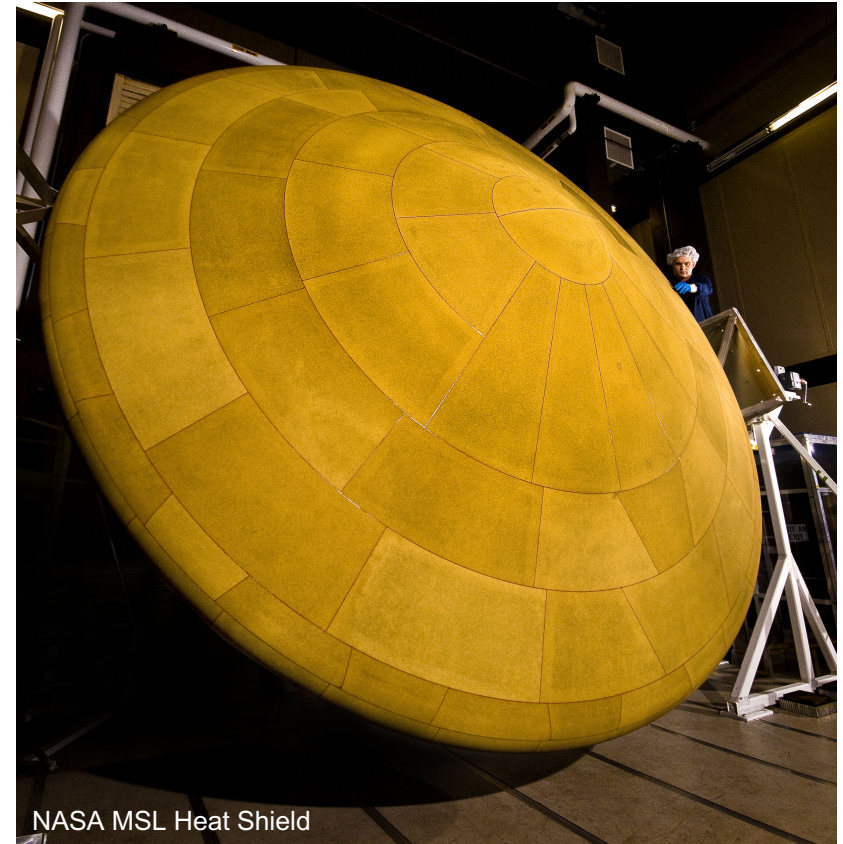
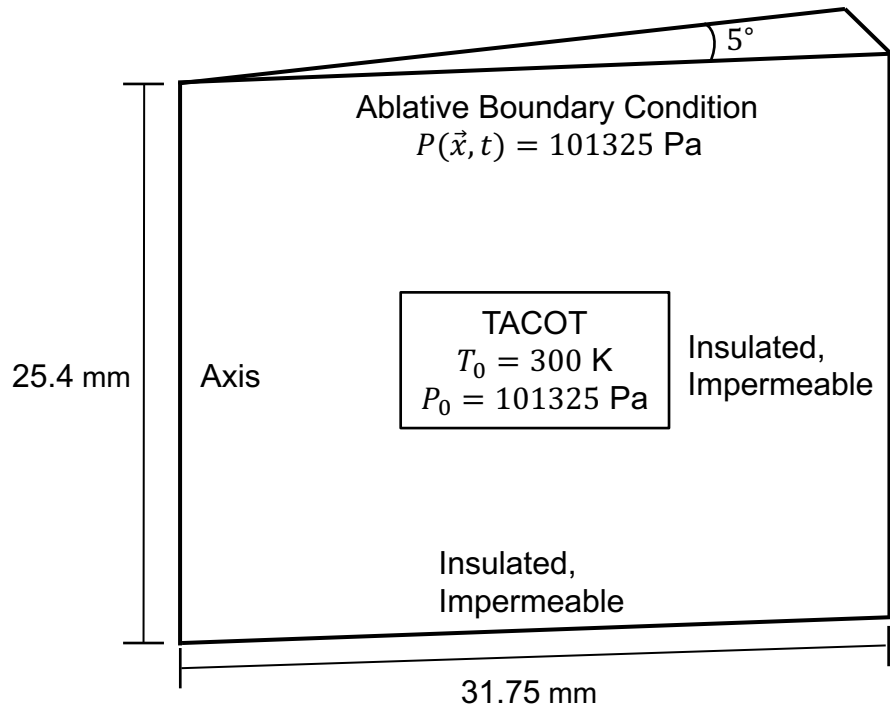
Axisymmetric Mini Arc Jet Case

Probe Locations



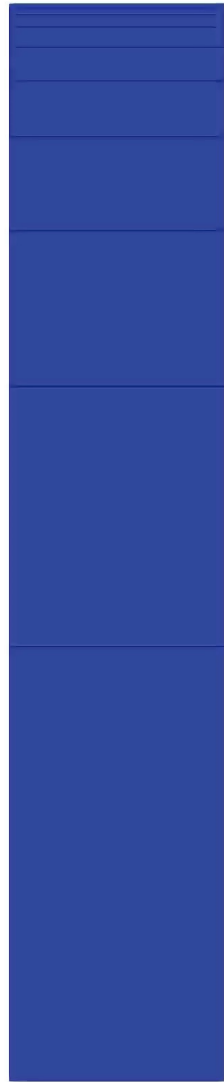
Probe	r (mm)	y (mm)	Probe	r (mm)	y (mm)
1	0	1.4	7	20	1.4
2	0	2.4	8	20	2.4
3	0	3.4	9	20	3.4
4	0	4.4	10	20	4.4
5	0	5.4	11	20	5.4
6	0	10.4	12	20	10.4



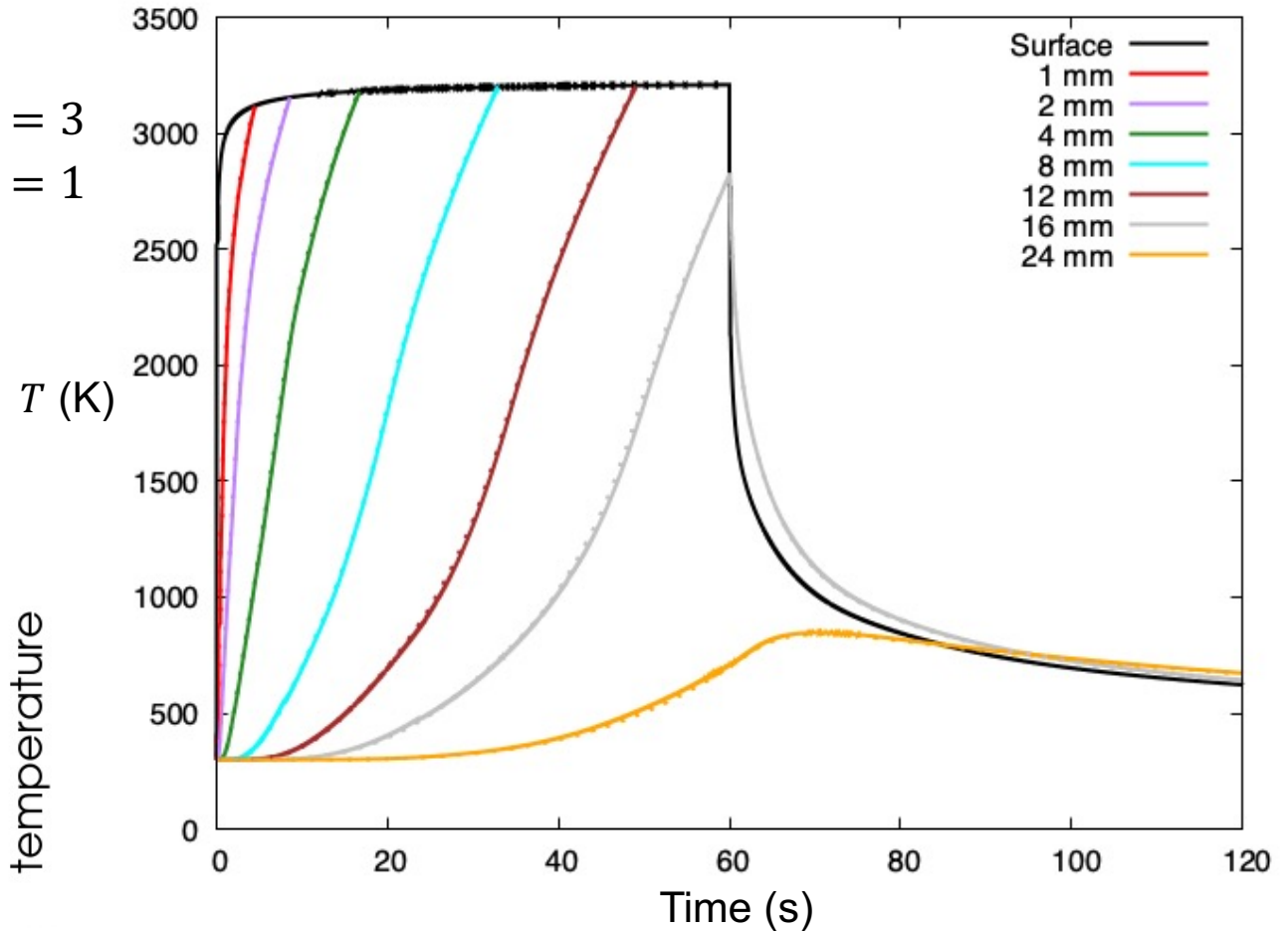
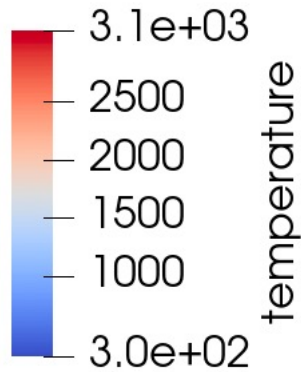


High-Order Solutions

10 element $p = 3$ solution compared to 100 element $p = 1$
3.67x faster time to solution



— $p = 3$
- - - $p = 1$



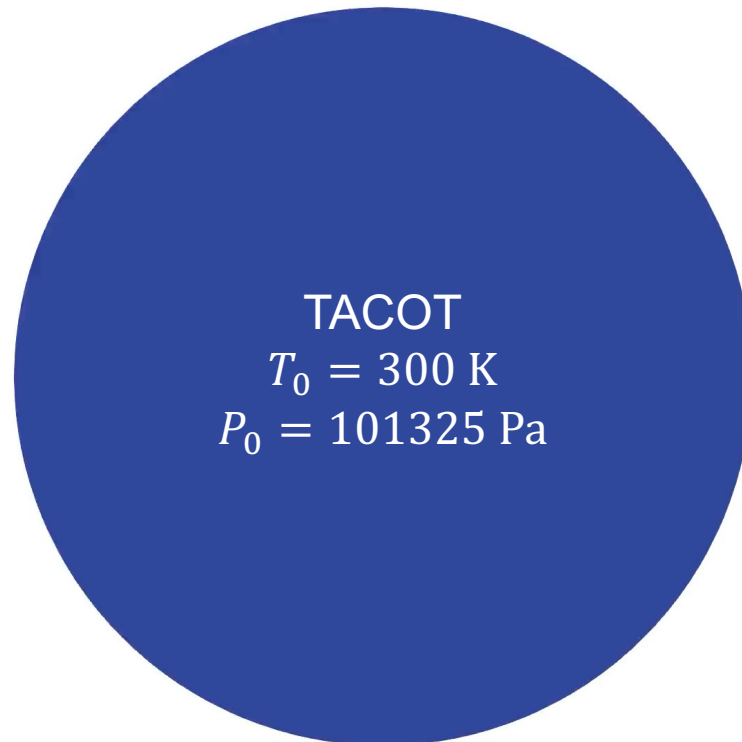
High-Order Meshes

Boundary Condition:

$$\rho U_e C_H = \frac{0.3(R - x)}{2R}$$

$$h_e = 1.5 \frac{\text{MJ}}{\text{kg}}$$

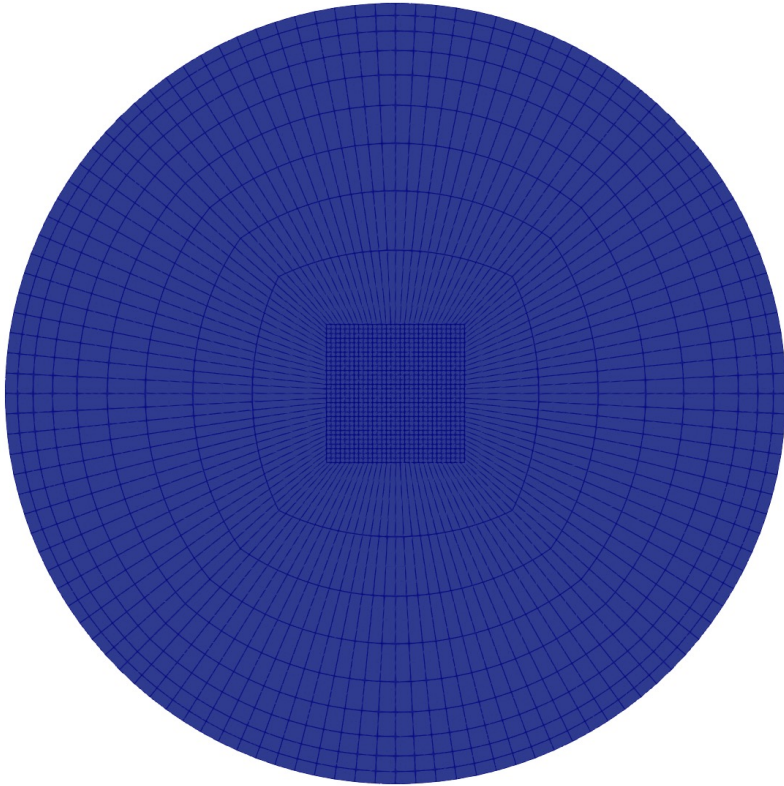
$$P = 101325 \text{ Pa}$$



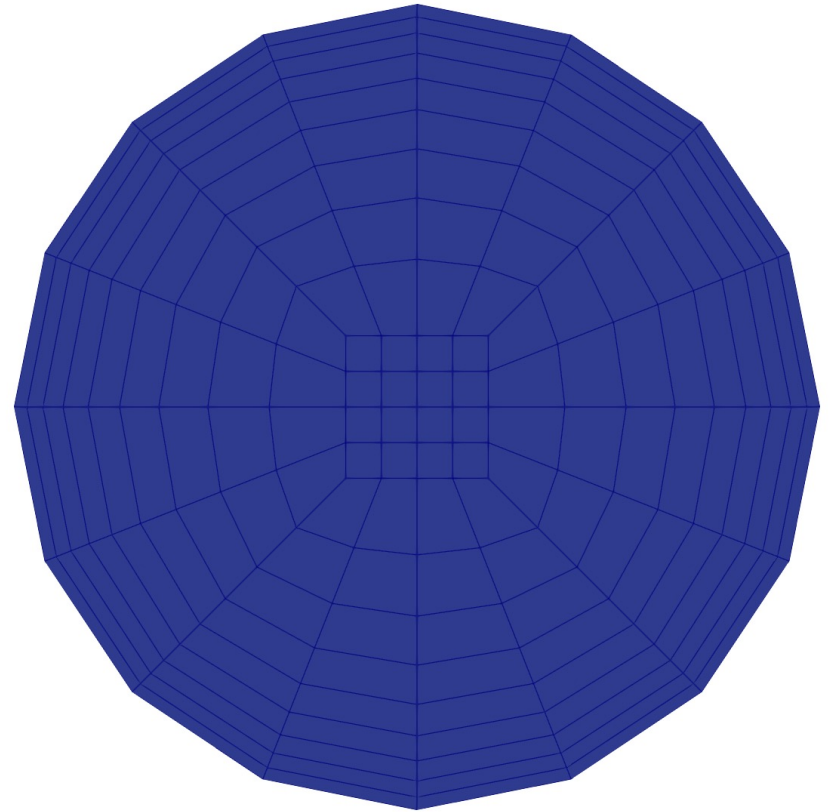
$$R = 20 \text{ mm}$$



High-Order Meshes



Refined Linear Mesh
2100 Elements

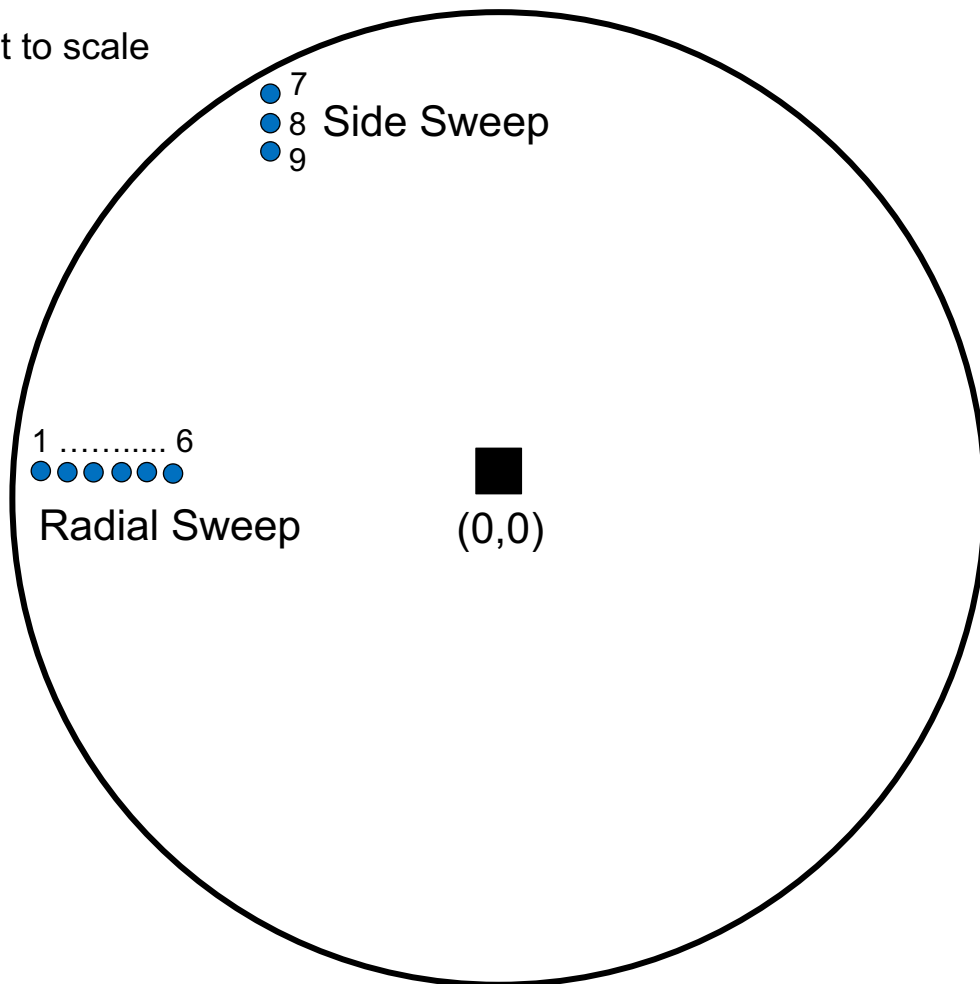


Coarse Mesh (Linear)
176 Elements



High-Order Meshes

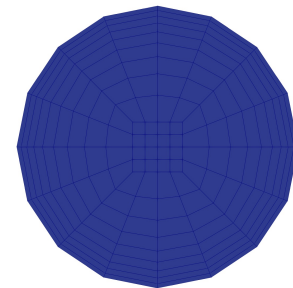
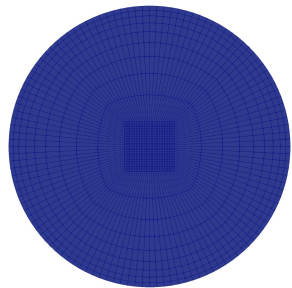
Not to scale



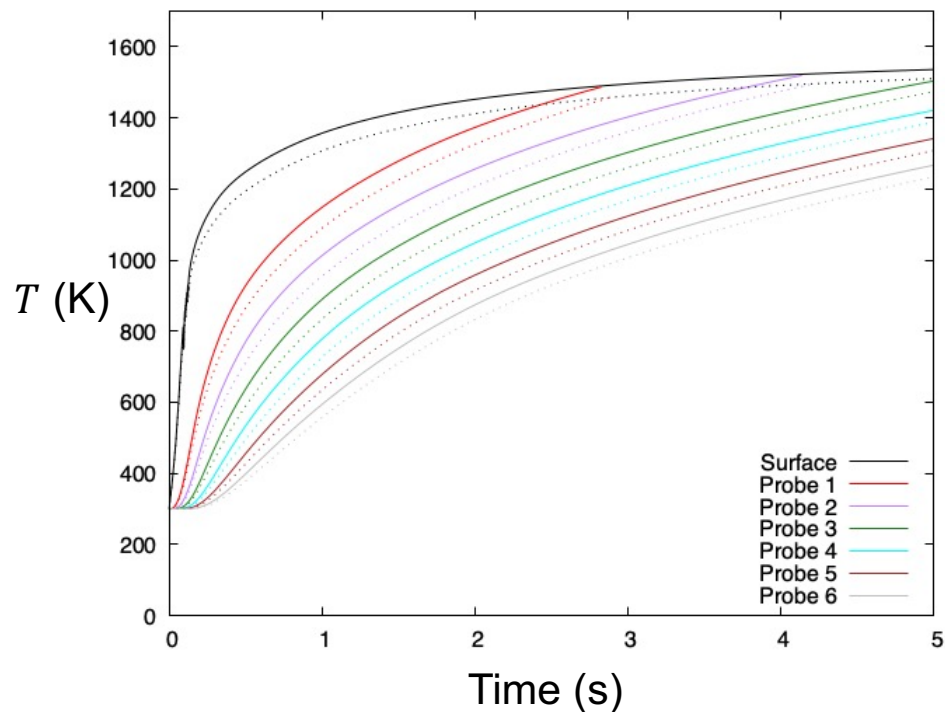
Probe	x (mm)	y(mm)
1	-9.50	0.0
2	-9.25	0.0
3	-9.00	0.0
4	-8.75	0.0
5	-8.50	0.0
6	-8.25	0.0
7	-7.07	6.82
8	-7.07	6.57
9	-7.07	6.32



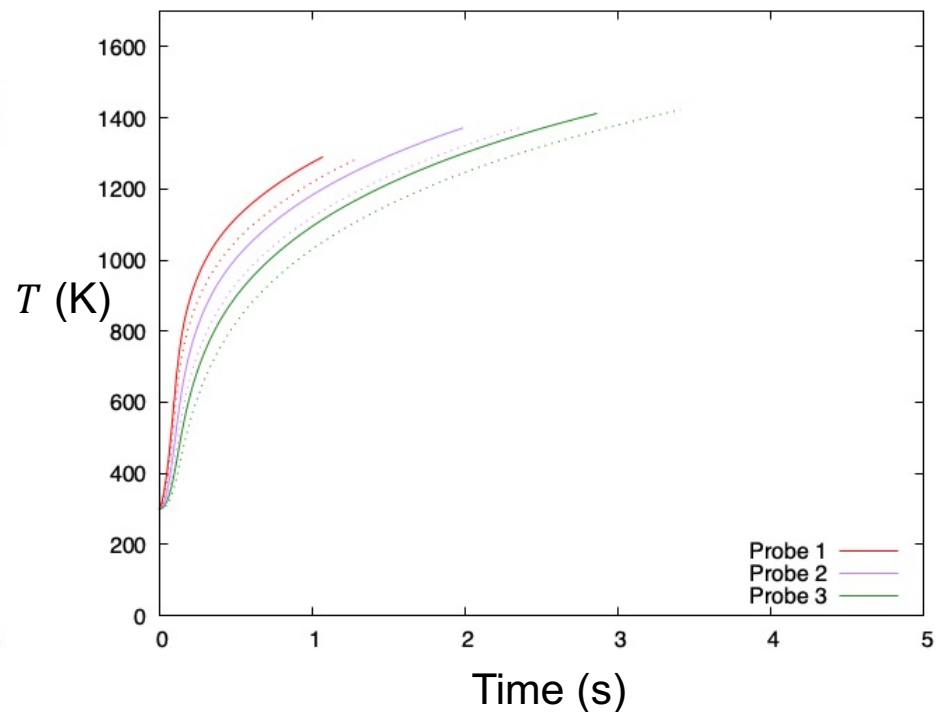
High-Order Meshes



Radial Sweep



Side Sweep

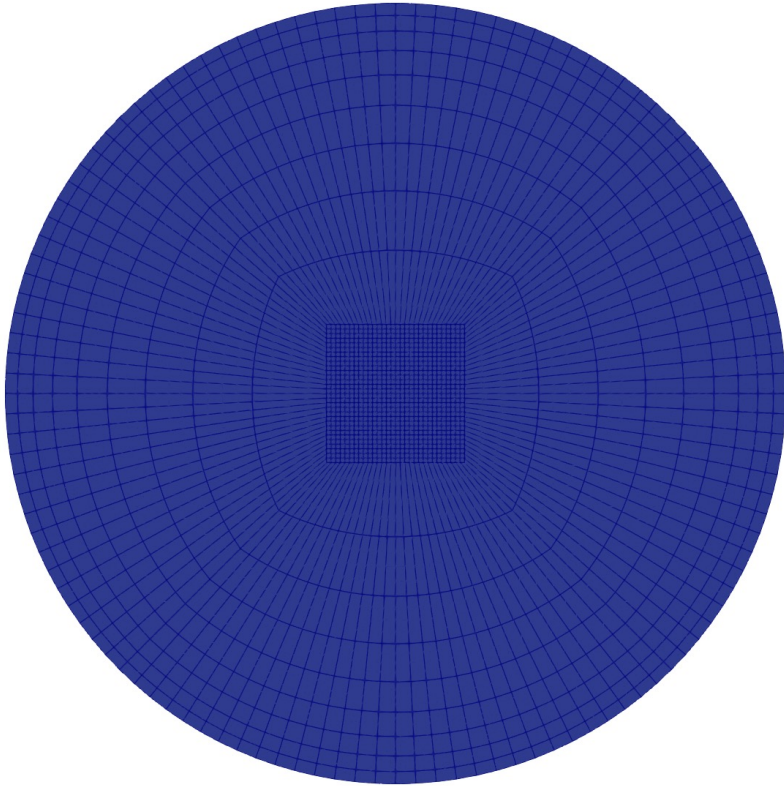


— Coarse Linear Mesh
- - - Refined Linear Mesh

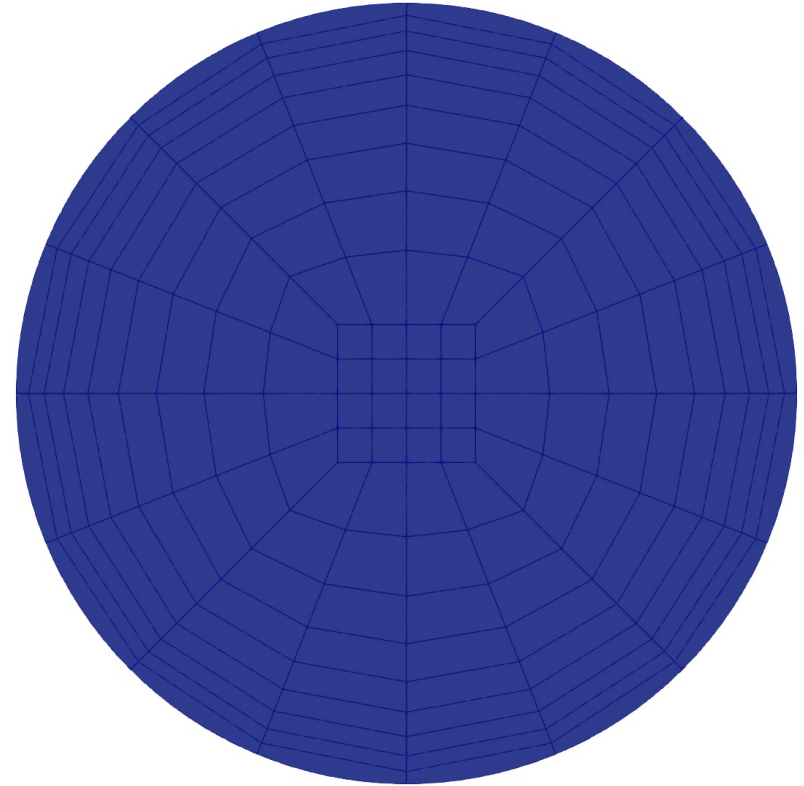
Both use $p = 2$ for internal solution



High-Order Meshes



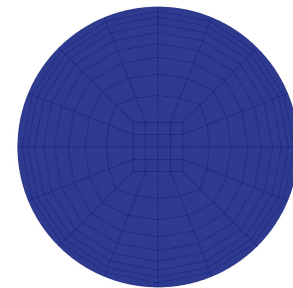
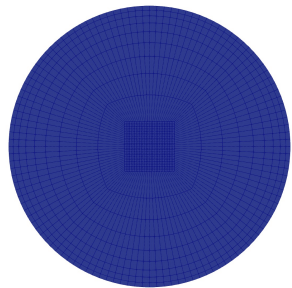
Refined Linear Mesh
2100 Elements



Coarse Mesh (Quadratic)
176 Elements

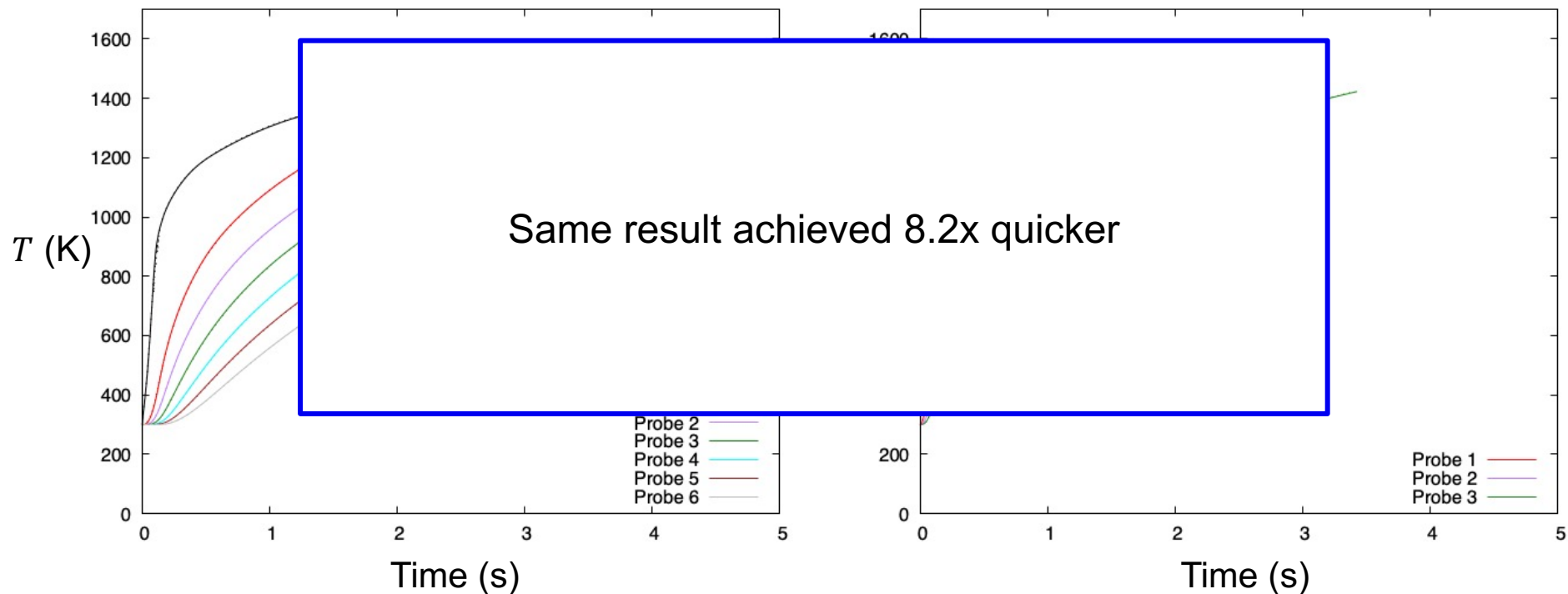


High-Order Meshes



Radial Sweep

Side Sweep



— Coarse Quadratic Mesh
 - - - Refined Linear Mesh

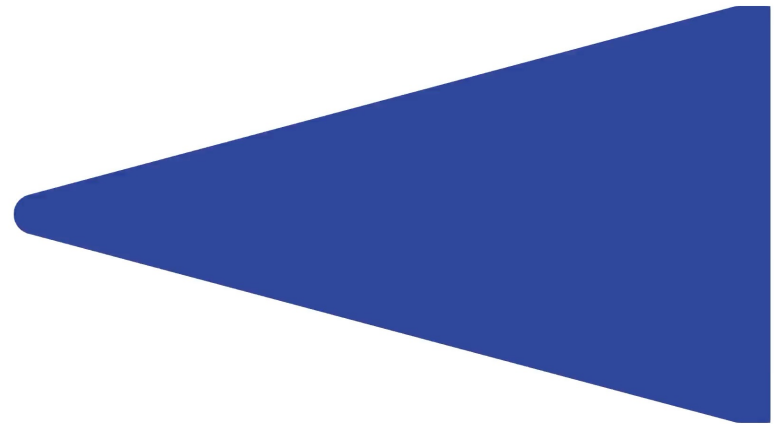
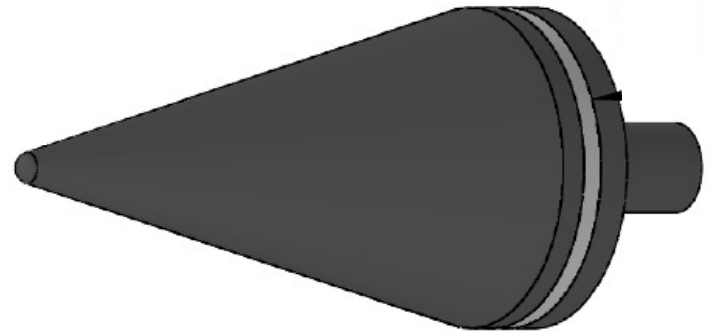
Both use $p = 2$ for internal solution



Future Work

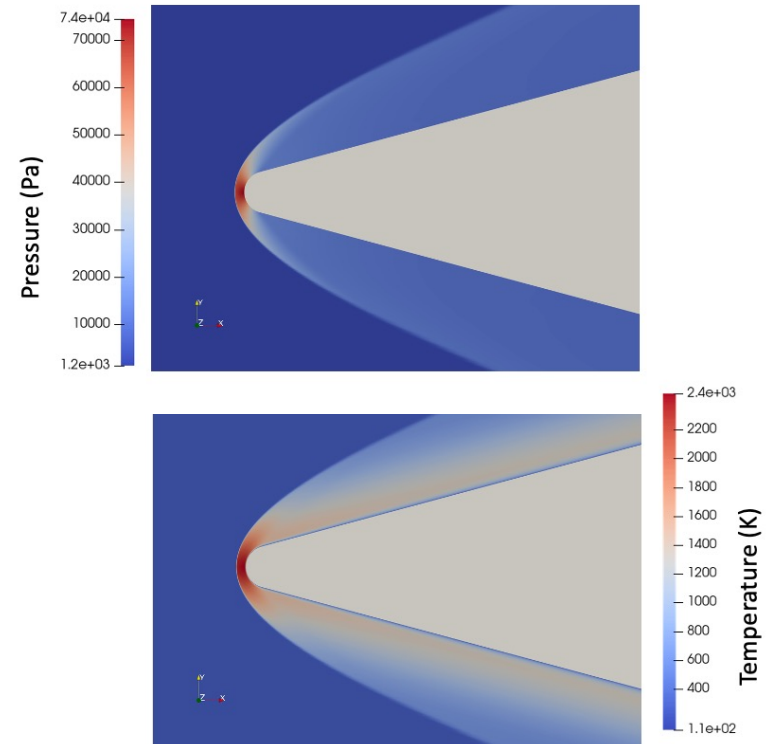
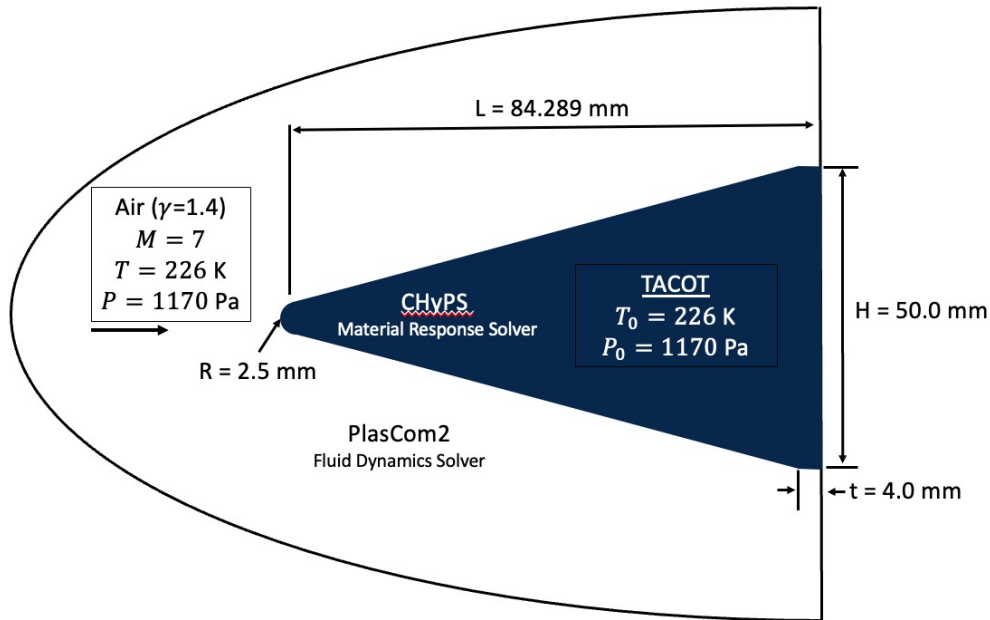
Coupling with External Flow Solvers

Mini-Arc Jet Case



Future Work

Coupling with External Flow Solvers

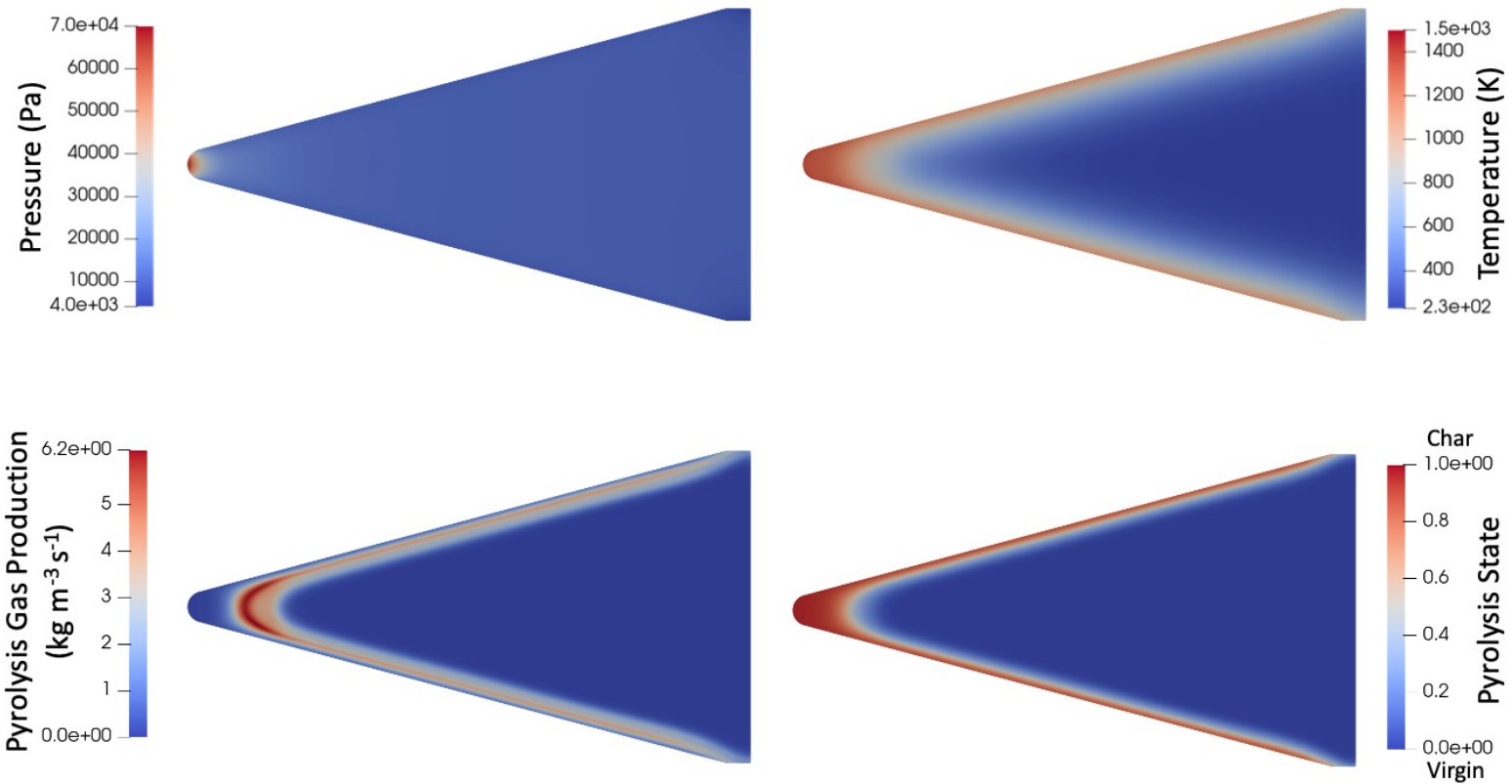


Use PlasCom2 to generate the external flow solution

Future Work

Coupling with External Flow Solvers

Material Response after 10 Seconds

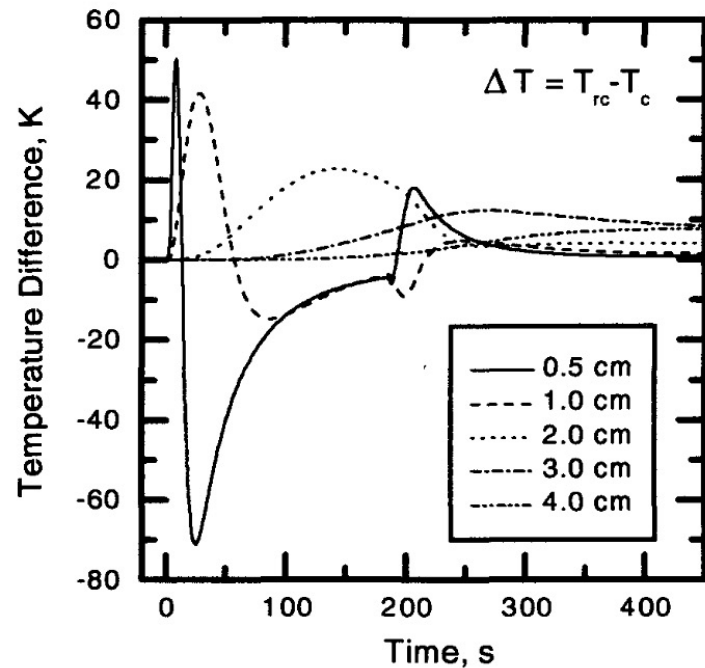
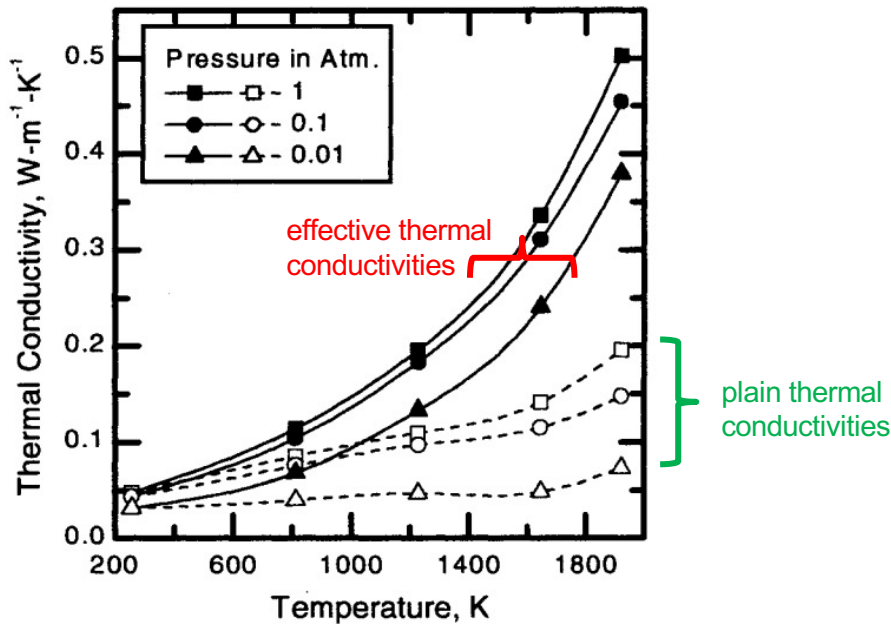


Future Work

Internal Radiation using P1 Approximation

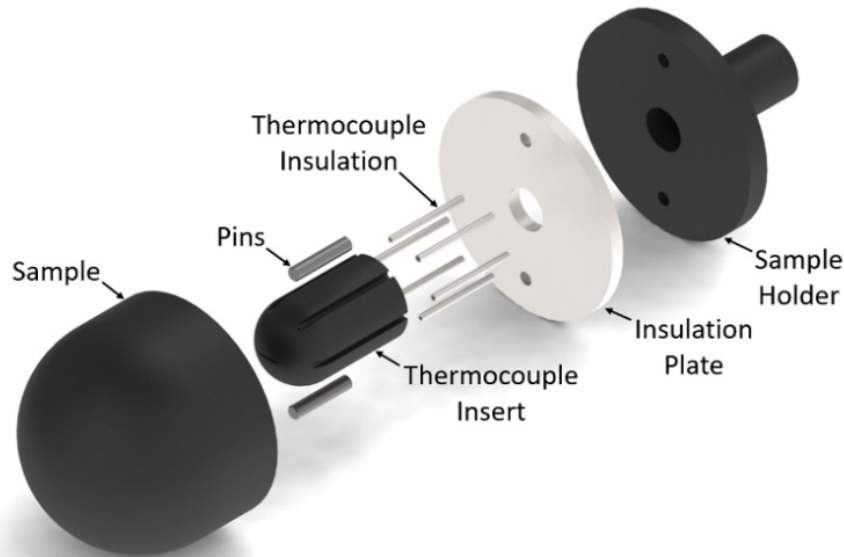
$$\frac{1}{3\kappa} \nabla \cdot \left(\frac{1}{\beta - \frac{A_1 \sigma_s}{3}} \nabla G \right) - G = -4\pi I_b, \quad -\frac{2-\epsilon}{\epsilon} \frac{2}{3\beta - A_1 \sigma_s} \hat{\mathbf{n}} \cdot \nabla G + G = 4\pi I_{bw}$$

$$\mathbf{q} = -\frac{1}{3\beta - A_1 \sigma_s} \nabla G$$

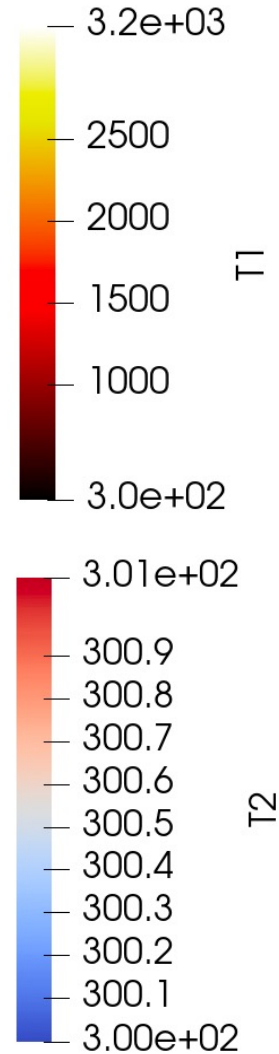


Future Work

Insulation Estimation for Experimental Samples



Cross-section after
10 s of heating



Conclusions

- Design of next-generation thermal protection systems and hypersonic vehicles will benefit greatly from computational studies
- Macroscopic volume averaged approach to simulating reactive porous materials aimed at enabling full-vehicle simulations
- Code-to-code comparison has been performed against NASA's PATO code
- Continued work will be done to leverage use of high-order solutions

