The University of Texas at Austin
Oden Institute for Computational Engineering and Sciences

Axisymmetric MFEM-based solvers for the compressible Navier-Stokes equations and other problems

Raphaël Zanella

MFEM Seminar · March 1, 2022

ODEN INSTITUTE
PECOS Predictive Engineering & Computational Science

Outline

Motivation

Laplacian solver

Heat equation solver

Compressible flow solver

Conclusion

Outline

Motivation

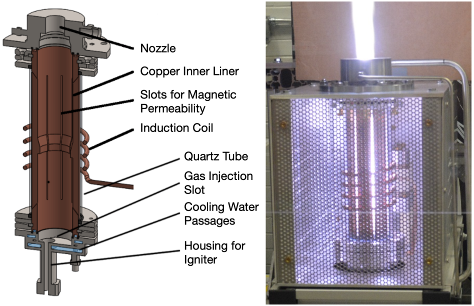
Laplacian solver

Heat equation solver

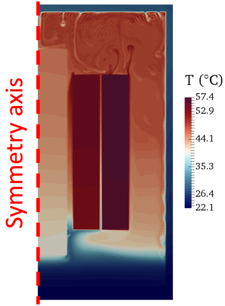
Compressible flow solver

Conclusion

Motivation for an axisymmetric model



Plasma torch



Transformer axisymmetric model

- System and external action roughly axisymmetric
- Non-axisymmetric effects expected to be small
- Highly accurate solution is not a priority (UQ, sensitivity analysis, ...)

$\rightarrow$ Axisymmetric modeling and **significant cut in the computational cost**

Outline

Motivation

Laplacian solver

Heat equation solver

Compressible flow solver

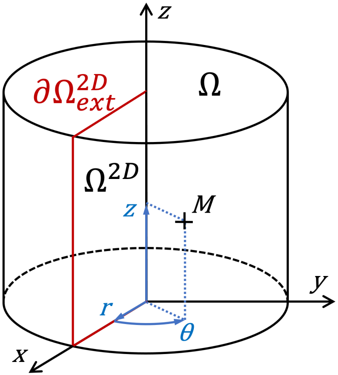
Conclusion

Problem description

$$\begin{cases} -\Delta u = f & \text{in } \Omega \\ u = u_b & \text{on } \partial\Omega \end{cases}$$

Ω: axisymmetric domain
u: unknown solution field
f: axisymmetric source term
u_b: axisymmetric boundary value

Axisymmetric approximation spaces



Notations
$\mathcal{T}_h$: mesh of Ω^{2D}
$p \in \mathbb{N}^*$: order of the polynomial approximation
$\partial\Omega^{2D}_{ext} = \partial\Omega \cap \overline{\Omega^{2D}}$

Trial space

$$V^{2D} = \left\{ v_h \in \mathcal{C}^0\left(\overline{\Omega^{2D}}; \mathbb{R}\right);\ v_{h|_K} \in \mathbb{P}_p,\ \forall K \in \mathcal{T}_h \right\}$$

$$V = \left\{ v_h \in \mathcal{C}^0\left(\overline{\Omega}; \mathbb{R}\right);\ \exists v_h^{2D} \in V^{2D};\ v_h(r,\theta,z) = v_h^{2D}(r,z),\ \forall (r,\theta,z) \right\}$$

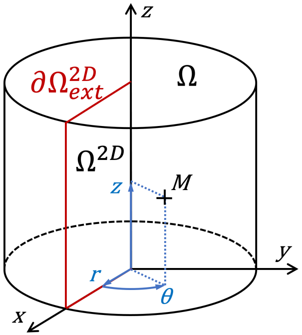
Test space

$$V_0^{2D} = \left\{ v_h \in V^{2D};\ v_h = 0 \text{ on } \partial\Omega^{2D}_{ext} \right\}$$

$$V_0 = \left\{ v_h \in \mathcal{C}^0\left(\overline{\Omega}; \mathbb{R}\right);\ \exists v_h^{2D} \in V_0^{2D};\ v_h(r,\theta,z) = v_h^{2D}(r,z),\ \forall (r,\theta,z) \right\}$$

Note: $\forall v_h \in V_0,\ v_h = 0$ on $\partial\Omega$

Axisymmetric weak formulation



$\forall F$ axisymmetric,

$$\int_{\Omega} F(r,\theta,z)dV = 2\pi \int_{\Omega^{2D}} rF(r,z)dS$$

Find $u_h \in V$ such that

$$\begin{cases} \int_{\Omega} \nabla u_h \cdot \nabla v_h dV = \int_{\Omega} f v_h dV, \ \forall v_h \in V_0 \\ u_h = u_{bh} \text{ on } \partial\Omega \end{cases}$$

u_{bh}: approximation of u_b in V

$\Leftrightarrow$

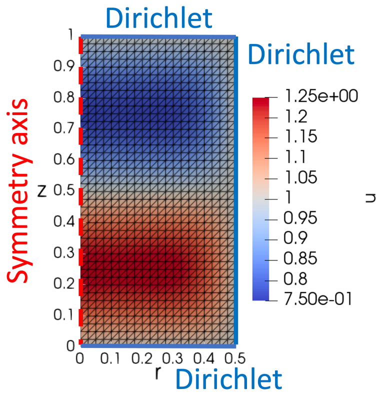
Find $u_h^{2D} \in V^{2D}$ such that

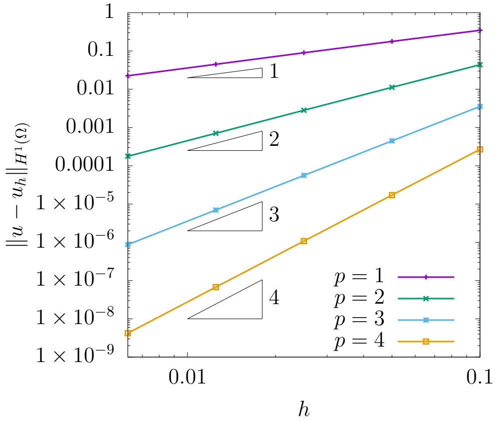
$$\begin{cases} \int_{\Omega^{2D}} r\nabla u_h^{2D} \cdot \nabla v_h^{2D} dS = \int_{\Omega^{2D}} r f v_h^{2D} dS, \ \forall v_h^{2D} \in V_0^{2D} \\ u_h^{2D} = u_{bh}^{2D} \text{ on } \partial\Omega^{2D}_{ext} \end{cases}$$

u_{bh}^{2D}: approximation of $u_{b|\Omega^{2D}}$ in V^{2D}

Convergence test on manufactured solution

Manufactured solution: $u(r,\theta,z) = (r^2(\sin(2\pi r) - 1) + 0.25)\sin(2\pi z) + 1$



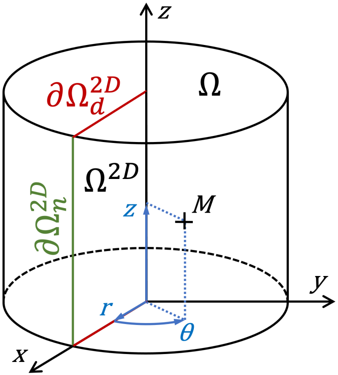


Addition of Neumann boundary conditions

$$\begin{cases} -\Delta u = f & \text{in } \Omega \\ u = u_b & \text{on } \partial\Omega_d \\ \nabla u \cdot \mathbf{n} = g & \text{on } \partial\Omega_n \end{cases}$$

Ω: axisymmetric domain
u: unknown solution field
f: axisymmetric source term
u_b: axisymmetric boundary value
g: axisymmetric boundary flux

Axisymmetric approximation spaces



Notations

$\mathcal{T}_h$: mesh of Ω^{2D}

$p \in \mathbb{N}^*$: order of the polynomial approximation

$\partial\Omega_d^{2D} = \partial\Omega_d \cap \overline{\Omega^{2D}}$, $\partial\Omega_n^{2D} = \partial\Omega_n \cap \overline{\Omega^{2D}}$

Trial space

$$V^{2D} = \left\{ v_h \in \mathcal{C}^0\left(\overline{\Omega^{2D}}; \mathbb{R}\right); \; v_{h|_K} \in \mathbb{P}_p, \; \forall K \in \mathcal{T}_h \right\}$$

$$V = \left\{ v_h \in \mathcal{C}^0\left(\overline{\Omega}; \mathbb{R}\right); \; \exists v_h^{2D} \in V^{2D}; \; v_h(r, \theta, z) = v_h^{2D}(r, z), \; \forall (r, \theta, z) \right\}$$

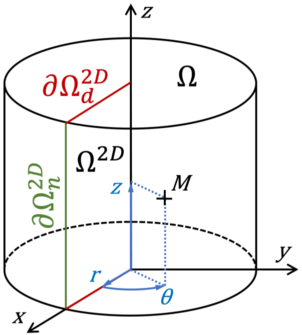
Test space

$$V_0^{2D} = \left\{ v_h \in V^{2D}; \; v_h = 0 \text{ on } \partial\Omega_d^{2D} \right\}$$

$$V_0 = \left\{ v_h \in \mathcal{C}^0\left(\overline{\Omega}; \mathbb{R}\right); \; \exists v_h^{2D} \in V_0^{2D}; \; v_h(r, \theta, z) = v_h^{2D}(r, z), \; \forall (r, \theta, z) \right\}$$

Note: $\forall v_h \in V_0$, $v_h = 0$ on $\partial\Omega_d$

Axisymmetric weak formulation



$\forall F$ axisymmetric,

$$\int_{\partial\Omega_n} F(r,\theta,z)dS = 2\pi \int_{\partial\Omega_n^{2D}} rF(r,z)dL$$

Find $u_h \in V$ such that

$$\begin{cases} \int_\Omega \nabla u_h \cdot \nabla v_h dV = \int_\Omega f v_h dV + \int_{\partial\Omega_n} g v dS, \ \forall v_h \in V_0 \\ u_h = u_{bh} \text{ on } \partial\Omega_d^{2D} \end{cases}$$

u_{bh}: approximation of u_b in V

$\Leftrightarrow$

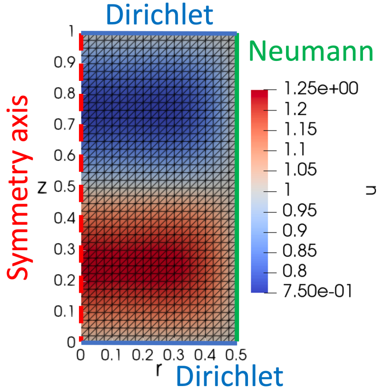
Find $u_h^{2D} \in V^{2D}$ such that

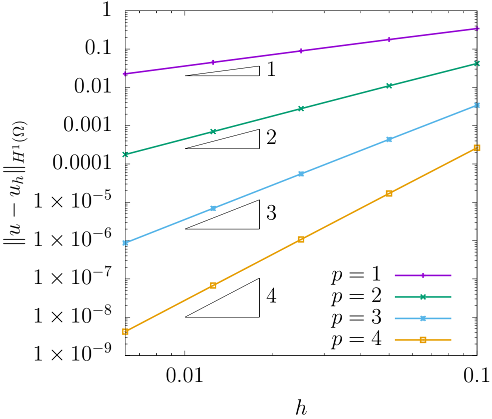
$$\begin{cases} \int_{\Omega^{2D}} r\nabla u_h^{2D} \cdot \nabla v_h^{2D} dS = \int_{\Omega^{2D}} rf v_h^{2D} dS + \int_{\partial\Omega_n^{2D}} rg v^{2D} dL, \ \forall v_h^{2D} \in V_0^{2D} \\ u_h^{2D} = u_{bh}^{2D} \text{ on } \partial\Omega_d^{2D} \end{cases}$$

u_{bh}^{2D}: approximation of $u_{b|\Omega^{2D}}$ in V^{2D}

Convergence test on manufactured solution

Manufactured solution: $u(r, \theta, z) = (r^2(\sin(2\pi r) - 1) + 0.25)\sin(2\pi z) + 1$





Outline

Motivation

Laplacian solver

Heat equation solver

Compressible flow solver

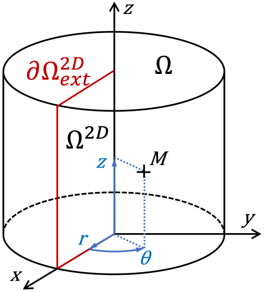
Conclusion

Problem description

$$\begin{cases} \partial_t u - \nabla\cdot(\kappa \nabla u) = f & \text{in } \Omega \times [0,T] \\ u = 0 & \text{on } \partial\Omega \times [0,T] \\ u_{|t=0} = u_0 & \text{in } \Omega \end{cases}$$

Ω: axisymmetric domain
u: unknown solution field
κ: diffusivity parameter
f: axisymmetric source term
u_0: axisymmetric initial condition

Axisymmetric weak formulation



$$V = \left\{ v_h \in \mathcal{C}^0\left(\overline{\Omega};\mathbb{R}\right);\ \exists v_h^{2D} \in V^{2D};\ v_h(r,\theta,z) = v_h^{2D}(r,z),\ \forall (r,\theta,z) \right\}$$

$$V^{2D} = \left\{ v_h \in \mathcal{C}^0\left(\overline{\Omega^{2D}};\mathbb{R}\right);\ v_h|_K \in \mathbb{P}_p,\ \forall K \in \mathcal{T}_h,\ \text{and } v_h = 0 \text{ on } \partial\Omega_{ext}^{2D} \right\}$$

$p \in \mathbb{N}^*$: order of the polynomial approximation, $\mathcal{T}_h$: mesh of Ω^{2D}

Find $u_h \in \mathcal{C}^1([0,T];V)$ such that

$$\begin{cases} \int_\Omega \frac{du_h}{dt}(t) v_h dV + \int_\Omega \kappa \nabla u_h(t)\cdot\nabla v_h dV = \int_\Omega f(t) v_h dV,\ \forall t \in [0,T],\ \forall v_h \in V \\ u_h(0) = u_{0h} \in V \end{cases}$$

$\Leftrightarrow$ Find $u_h^{2D} \in \mathcal{C}^1([0,T];V^{2D})$ such that

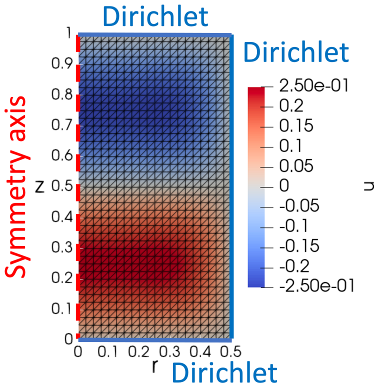
$$\begin{cases} \int_{\Omega^{2D}} \frac{du_h^{2D}}{dt}(t) v_h^{2D} r dS + \int_{\Omega^{2D}} \kappa \nabla u_h^{2D}(t)\cdot\nabla v_h^{2D} r dS = \int_{\Omega^{2D}} f(t) v_h^{2D} r dS, \\ \qquad \forall t \in [0,T],\ \forall v_h^{2D} \in V^{2D} \\ u_h^{2D}(0) = u_{0h}^{2D} \in V^{2D} \end{cases}$$

$\forall F$ axisymmetric,

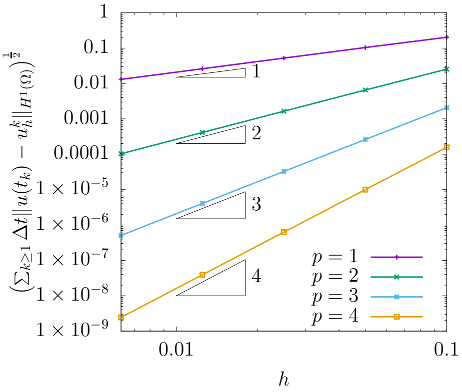
$$\int_\Omega F(r,\theta,z) dV = 2\pi \int_{\Omega^{2D}} r F(r,z) dS$$

Mesh size convergence test

Manufactured solution: $u(r,\theta,z) = ((r^2(\sin(2\pi r) - 1) + 0.25)\sin(2\pi z) + 1)t$



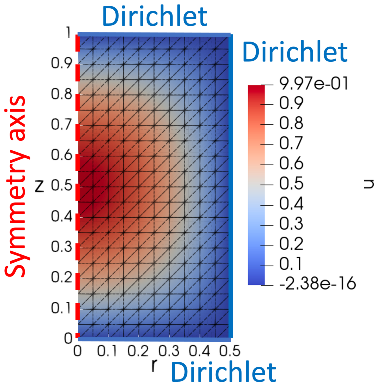
$t = 1$



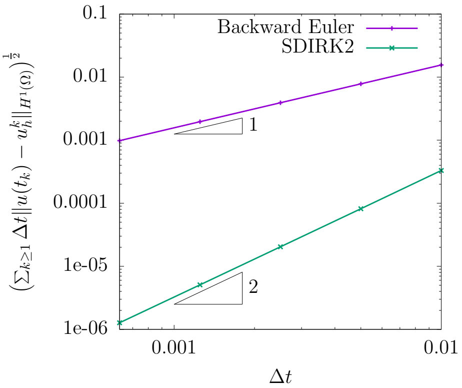
Backward Euler, $\Delta t = 0.01$, $T = 1$

Time step convergence test

Manufactured solution: $u(r,\theta,z) = 4\left(1-\left(\frac{r}{0.5}\right)^2\right) z(1-z)\cos(2\pi t)$



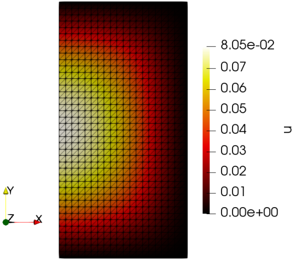
$t = 1$



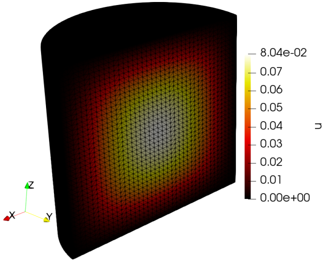
$h = 0.05,\ p = 4,\ T = 1$

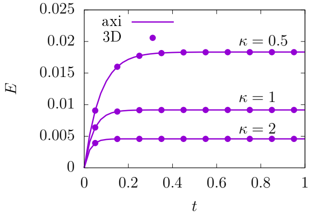
Axisymmetric versus 3D formulation I

Axisymmetric computation
Triangular mesh

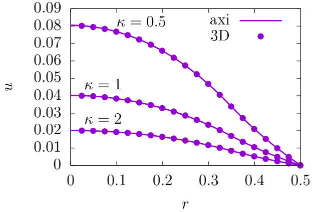


3D computation
Tetrahedral mesh (same h)

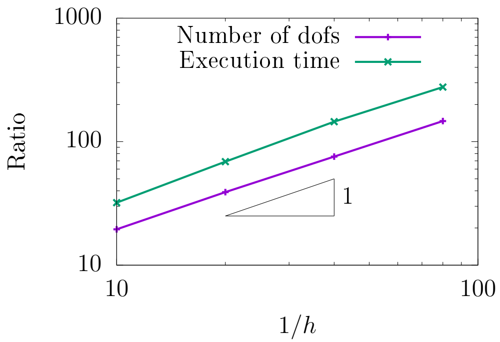




$E = \int_\Omega u dV$



Axisymmetric versus 3D formulation II



Quasi-identical results but axisymmetric code much faster (speedup $\propto 1/h$) due to the use of a 2D mesh instead of a 3D mesh

Outline

Motivation

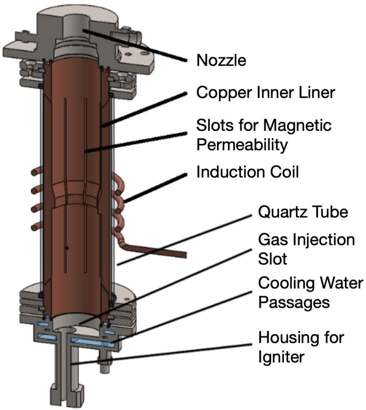
Laplacian solver

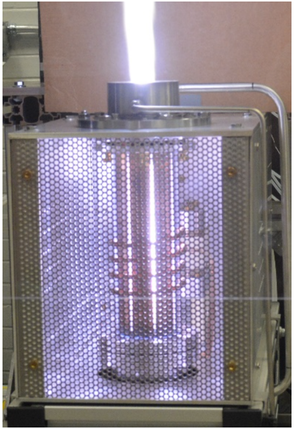
Heat equation solver

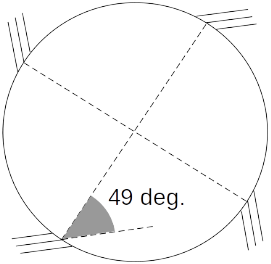
Compressible flow solver

Conclusion

Motivation: air flow in a plasma torch







System roughly axisymmetric
Gas injected tangentially
$\rightarrow$ Axisymmetric model taking into account u_θ

Governing equations I

Compressible Navier-Stokes equations in cylindrical coordinates (r, θ, z) with $\frac{\partial}{\partial \theta} = 0$:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial r \rho u_r}{\partial r} + \frac{\partial \rho u_z}{\partial z} = 0$$

$$\frac{\partial \rho u_r}{\partial t} + \frac{\partial \rho u_r u_r}{\partial r} + \frac{1}{r}(\rho u_r u_r - \rho u_\theta u_\theta) + \frac{\partial \rho u_r u_z}{\partial z} = - \frac{\partial p}{\partial r} + \frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}(\tau_{rr} - \tau_{\theta\theta}) + \frac{\partial \tau_{rz}}{\partial z}$$

$$\frac{\partial \rho u_\theta}{\partial t} + \frac{\partial \rho u_\theta u_r}{\partial r} + \frac{2}{r}\rho u_\theta u_r + \frac{\partial \rho u_\theta u_z}{\partial z} = \frac{\partial \tau_{\theta r}}{\partial r} + \frac{2}{r}\tau_{\theta r} + \frac{\partial \tau_{\theta z}}{\partial z}$$

$$\frac{\partial \rho u_z}{\partial t} + \frac{\partial \rho u_z u_r}{\partial r} + \frac{1}{r}\rho u_z u_r + \frac{\partial \rho u_z u_z}{\partial z} = - \rho g - \frac{\partial p}{\partial z} + \frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\tau_{zr} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\begin{aligned}\frac{\partial \rho E}{\partial t} + \frac{1}{r}\frac{\partial r \rho E u_r}{\partial r} + \frac{\partial \rho E u_z}{\partial z} = & - \rho g u_z + \frac{1}{r}\frac{\partial r((-p + \tau_{rr})u_r + \tau_{r\theta} u_\theta + \tau_{rz} u_z))}{\partial r} \\ & + \frac{\partial \tau_{zr} u_r + \tau_{z\theta} u_\theta + (-p + \tau_{zz}) u_z}{\partial z} - \frac{1}{r}\frac{\partial r q_r}{\partial r} - \frac{\partial q_z}{\partial z}\end{aligned}$$

ρ density, (u_r, u_θ, u_z) velocity components, p pressure, g gravity, $[\tau]$ viscous stress tensor, $(q_r, 0, q_z)$ heat flux vector components, $E = e + \frac{u^2}{2}$ total energy per unit mass (e internal energy)

Governing equations II

Ideal gas equation of state: $p = \rho R T$, R specific gas constant, T temperature, $h = e + \frac{p}{\rho}$ enthalpy per unit mass

$$e = c_v T, \qquad h = c_p T, \qquad R = c_p - c_v$$

c_v specific heat at constant volume, c_p specific heat at constant pressure

Viscous stress tensor components:

$$\tau_{rr} = \frac{2\eta}{3}\left(2\frac{\partial u_r}{\partial r} - \frac{u_r}{r} - \frac{\partial u_z}{\partial z}\right), \qquad \tau_{r\theta} = \eta\left(-\frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r}\right), \qquad \tau_{rz} = \eta\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right),$$

$$\tau_{\theta\theta} = \frac{2\eta}{3}\left(\frac{2u_r}{r} - \frac{\partial u_r}{\partial r} - \frac{\partial u_z}{\partial z}\right), \qquad \tau_{\theta z} = \eta\frac{\partial u_\theta}{\partial z}, \qquad \tau_{zz} = \frac{2\eta}{3}\left(2\frac{\partial u_z}{\partial z} - \frac{u_r}{r} - \frac{\partial u_r}{\partial r}\right),$$

$$\tau_{\theta r} = \tau_{r\theta}, \qquad \tau_{zr} = \tau_{rz}, \qquad \tau_{z\theta} = \tau_{\theta z}$$

Heat flux vector components:

$$q_r = -\lambda\frac{\partial T}{\partial r}, \qquad q_z = -\lambda\frac{\partial T}{\partial z}$$

Governing equations III

Viscosity law:

$$\eta(T) = \eta_{ref} \left(\frac{T}{T_{ref}} \right)^n$$

η_{ref} dynamic viscosity at a reference temperature T_{ref}, n constant coefficient

Thermal conductivity law:

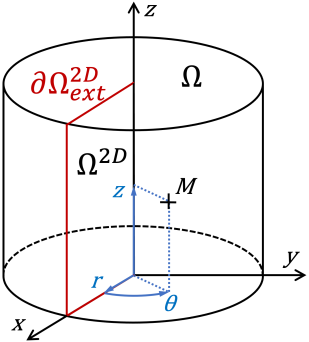
$$\lambda(T) = \frac{\eta(T) c_p}{P_r}$$

P_r Prandtl number, considered constant

Boundary conditions

- Isothermal wall: $T(t) = T_0$, $\mathbf{u}(t) = \mathbf{u}_0$
- Inlet: $\mathbf{u}(t) = \mathbf{u}_0$, $T(t) = T_0$
- Outlet: $p(t) = p_0$
- Axis: $u_r(t) = u_\theta(t) = 0$

Axisymmetric finite element spaces



Notations:
$\mathcal{T}_h$ mesh of Ω^{2D} with characteristic mesh size h
K cell of $\mathcal{T}_h$
$p \in \mathbb{N}^*$ order of the polynomial approximation

Trial space for ρ and ρE:

$$V = \left\{ v \in \mathcal{C}^0\left(\overline{\Omega}; \mathbb{R}\right); \exists v^{2D} \in V^{2D}; \quad v(r,\theta,z) = v^{2D}(r,z), \ \forall (r,\theta,z) \right\}$$

where

$$V^{2D} = \left\{ v \in \mathcal{C}^0\left(\overline{\Omega^{2D}}; \mathbb{R}\right); \ v_{|_K} \in \mathbb{P}_p, \forall K \in \mathcal{T}_h \right\}$$

Trial space for $\rho\mathbf{u}$:

$$\mathbf{V} = V^3$$

Test spaces for ρ, $\rho\mathbf{u}$, ρE:

$$V_{0,\rho}, \quad \mathbf{V}_0, \quad V_{0,\rho E}$$

Weak formulation

Find $\rho \in \mathcal{C}^1([0,t_f];V)$, $\rho\mathbf{u} \in \mathcal{C}^1([0,t_f];\mathbf{V})$ and $\rho E \in \mathcal{C}^1([0,t_f];V)$ satisfying the boundary conditions such that

$$\int_{\Omega^{2D}} \frac{d\rho}{dt} v r dS = \int_{\Omega^{2D}} \rho\mathbf{u} \cdot \nabla v r dS - \int_{\partial\Omega^{2D}_{ext}} v\rho\mathbf{u} \cdot \mathbf{n} r dL, \ \forall v \in V_{0,\rho}$$

$$\begin{aligned}\int_{\Omega^{2D}} \frac{d\rho\mathbf{u}}{dt} \cdot \mathbf{v} r dS = &\int_{\Omega^{2D}} (\rho\mathbf{u} \otimes \mathbf{u}) : \nabla \mathbf{v} r dS - \int_{\partial\Omega^{2D}_{ext}} ((\rho\mathbf{u} \otimes \mathbf{u}) \cdot \mathbf{v}) \cdot \mathbf{n} r dL \\ &- \int_{\Omega^{2D}} [\sigma] : \nabla \mathbf{v} r dS + \int_{\partial\Omega^{2D}_{ext}} ([\sigma] \cdot \mathbf{v}) \cdot \mathbf{n} r dL \\ &+ \int_{\Omega^{2D}} \rho\mathbf{g} \cdot \mathbf{v} r dS, \ \forall \mathbf{v} \in \mathbf{V}_0\end{aligned}$$

$$\begin{aligned}\int_{\Omega^{2D}} \frac{d\rho E}{dt} v r dS = &\int_{\Omega^{2D}} \rho E\mathbf{u} \cdot \nabla v r dS - \int_{\partial\Omega^{2D}_{ext}} v\rho E\mathbf{u} \cdot \mathbf{n} r dL \\ &- \int_{\Omega^{2D}} ([\sigma] \cdot \mathbf{u} - \mathbf{q}) \cdot \nabla v r dS + \int_{\partial\Omega^{2D}_{ext}} v([\sigma] \cdot \mathbf{u} - \mathbf{q}) \cdot \mathbf{n} r dL \\ &+ \int_{\Omega^{2D}} \rho\mathbf{u} \cdot \mathbf{g} v r dS, \ \forall v \in V_{0,\rho E}\end{aligned}$$

Time integration

Matrix form of the weak formulation:

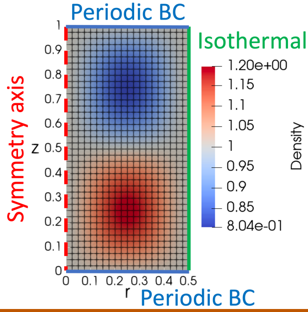
$$\begin{cases} \mathcal{M}\dfrac{d\mathcal{U}}{dt}(t) = \mathcal{R}(\mathcal{U}(t)), \ \forall t \in [0, t_f] \\ \mathcal{U}(0) = \mathcal{U}^0 \end{cases}$$

$\mathcal{M} \in \mathbb{R}^{5n_{dof} \times 5n_{dof}}$ mass matrix (n_{dof} number of degrees of freedom)
$\mathcal{R}$ nonlinear function of the dofs describing the flux terms and the gravity terms
$\mathcal{U}^0 \in \mathbb{R}^{5n_{dof}}$ dofs of the initial condition projected in V^5

Several explicit methods possible for time integration: forward Euler or Runge-Kutta of different orders

Convergence test on a manufactured solution

$$\begin{cases} \rho(r,z,t) = 1 + 50r^2(0.5-r)^2\sin(2\pi z)\cos(2\pi t) \\ u_r(r,z,t) = r^2\sin(2\pi r)\sin(2\pi z)\cos(2\pi t) \\ u_\theta(r,z,t) = r^2\sin(2\pi r)\sin(2\pi z)\cos(2\pi t) \\ u_z(r,z,t) = r^2(\cos(\pi r)\sin(2\pi z)\cos(2\pi t) - 1) + 0.25 \\ T(r,z,t) = 1 + r^2\cos(\pi r)\sin(2\pi z)\cos(2\pi t) \end{cases}$$



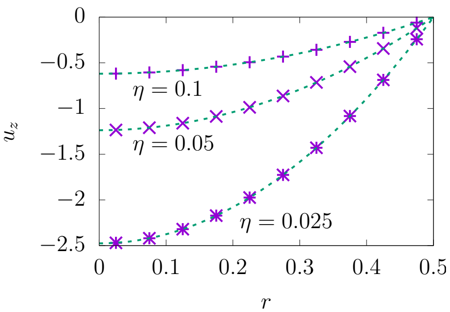
First order FE
2^{nd} order Runge-Kutta method
Fixed small time step $\tau = 5 \times 10^{-5}$
Errors at final time $t_f = 1$

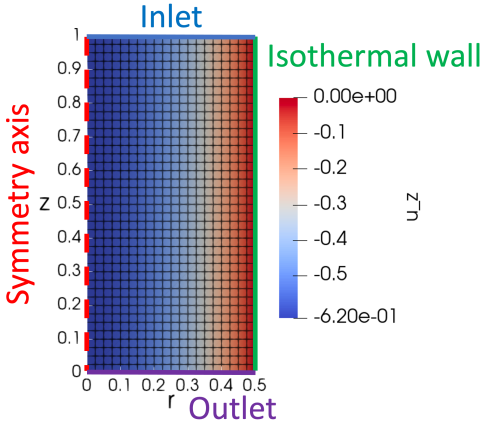
h	$\|U - U_{ex}\|_{L^2(\Omega)}$	COC
0.1	0.008600558	
0.05	0.0021620784	1.992
0.025	0.00054275361	1.994
0.0125	0.0001358782	1.998

$U = (\rho, \rho u_r, \rho u_\theta, \rho u_z, \rho E)$

Test: Poiseuille flow in a tube

$$
\begin{cases}
\rho(r,z,t) = \rho_0 \\
u_r(r,z,t) = u_\theta(r,z,t) = 0 \\
u_z(r,z,t) = -\dfrac{\rho_0 g}{4\eta}(R_0^2 - r^2) \\
T(r,z,t) = T_0
\end{cases}
$$

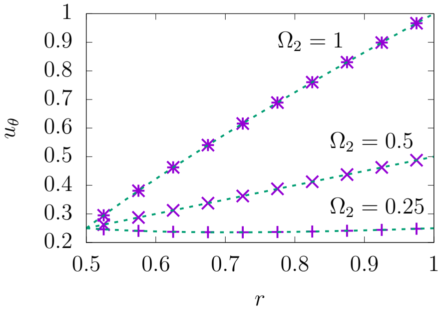


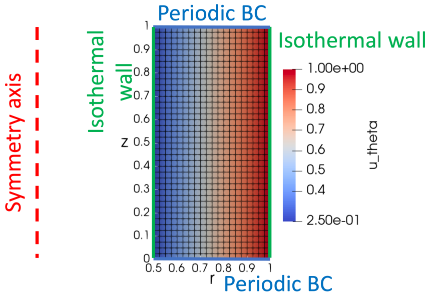


Test: Taylor-Couette flow

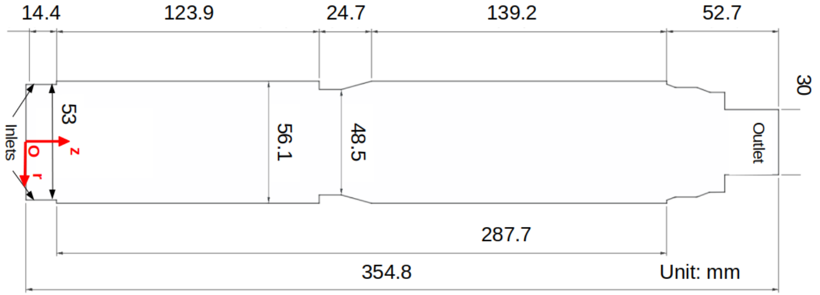
$$\begin{cases} \rho(r,z,t) = \rho_0 \\ u_r(r,z,t) = u_z(r,z,t) = 0, \;\; u_\theta(r,z,t) = Ar + \dfrac{B}{r} \\ T(r,z,t) = T_0 + \dfrac{B^2(r^2 - R_1^2) + r^2 R_1^2\left(A^2(r^2 - R_1^2) + 4AB\log\left(\frac{r}{R_1}\right)\right)}{2r^2 R_1^2 c_v(\gamma - 1)} \end{cases}$$

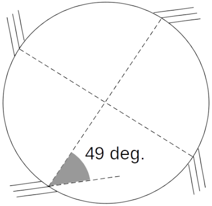
$$A = \frac{\Omega_2 R_2^2 - \Omega_1 R_1^2}{R_2^2 - R_1^2}, \quad B = \frac{(\Omega_1 - \Omega_2) R_1^2 R_2^2}{R_2^2 - R_1^2}$$





Air flow in a torch geometry: Setup





Top view of inlet channels and exit angle

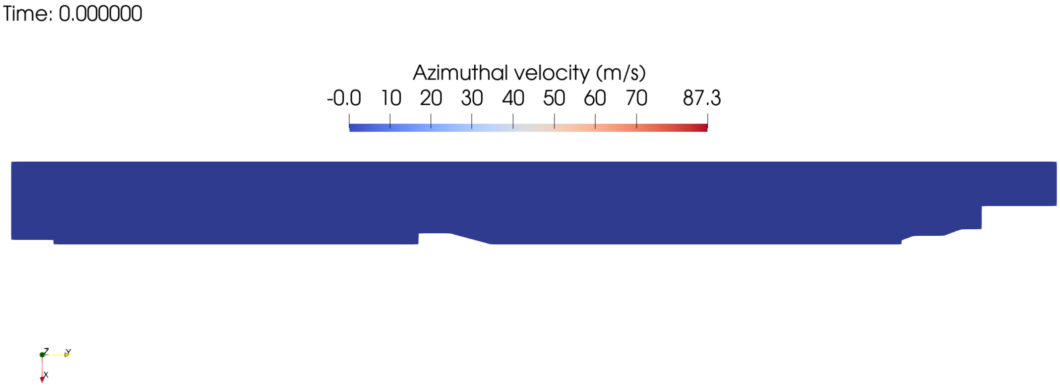
Inlets modeled by axisymmetric inlet preserving mass flow rate and tangential velocity

Scalability

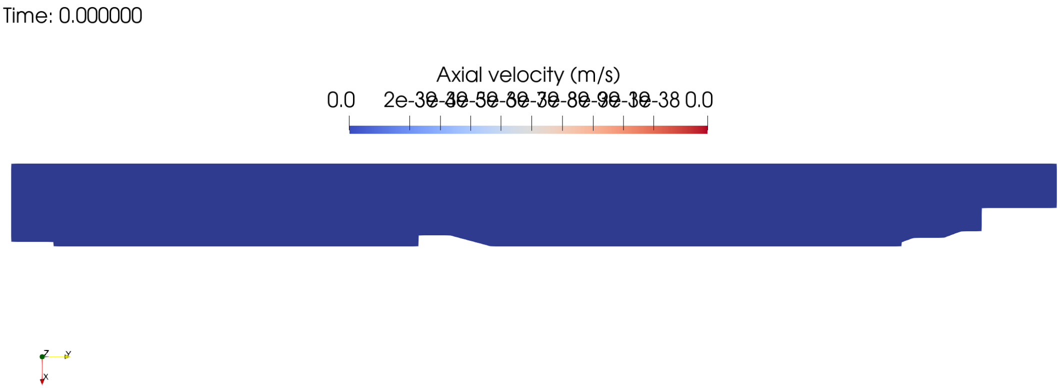
MPI proc.	Elapsed time (s)	Speed up	Scalability
1	6574.5672	1.0	1.0
36	237.12154	27.727	0.77
72	132.73101	49.533	0.688
108	88.398844	74.374	0.689
144	70.009964	93.909	0.652
180	58.943087	111.541	0.62
216	53.307597	123.333	0.571
252	52.826921	124.455	0.494
288	43.079631	152.614	0.53
324	41.369725	158.922	0.491
360	41.055994	160.137	0.445
396	37.644591	174.648	0.441

Mesh with 234187 nodes, 1170935 unknowns, 1000 iterations

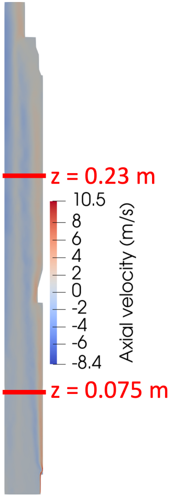
Air flow in a torch geometry: Simulation (u_θ)

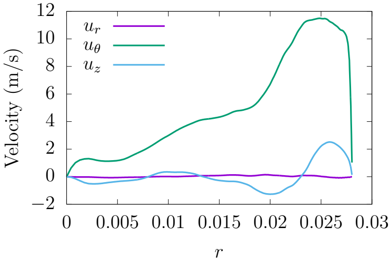


Air flow in a torch geometry: Simulation (u_z)

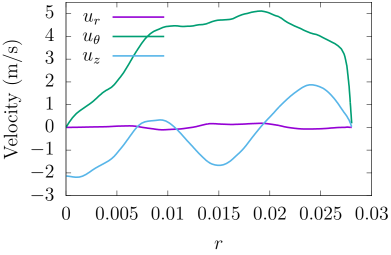


Time-averaged fields in the torch geometry





$z = 0.075$ m

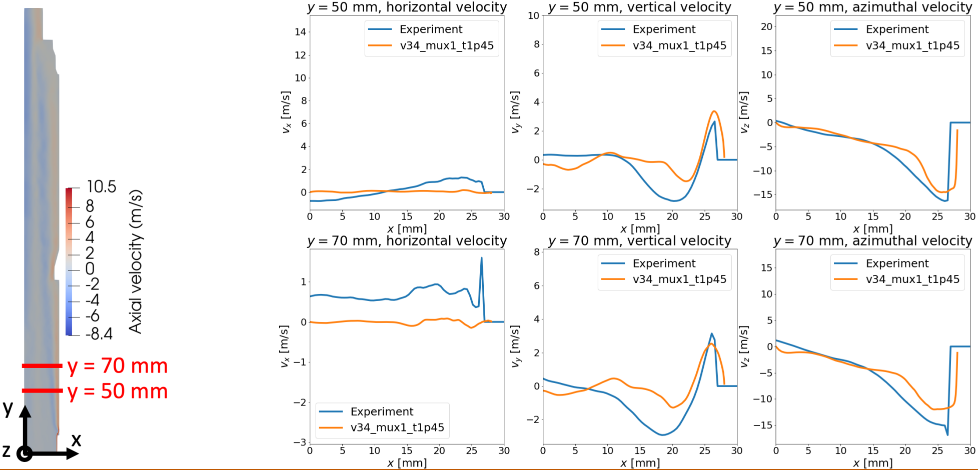


$z = 0.23$ m

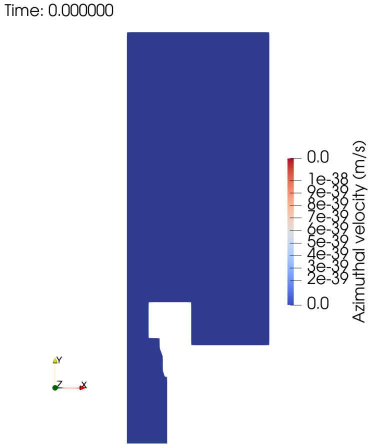
Flow localized close to the wall in the bottom compartment
Layers of upward / downward flow

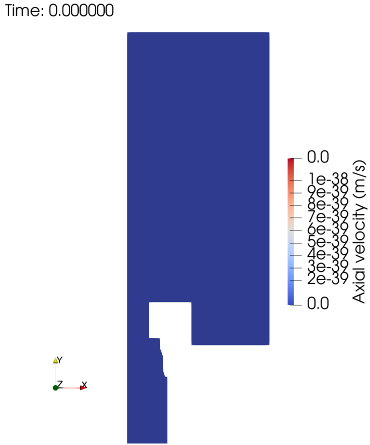
Comparison with experiments

Experiments: Dillon Ellender & Dan Fries, UT Austin



Inflow at the outlet





Outline

Motivation

Laplacian solver

Heat equation solver

Compressible flow solver

Conclusion

Conclusion

Summary

- Implementation of axisymmetric solvers for the Laplaction problem, the heat equation and the compressible Navier-Stokes equations
- Simple modifications are needed to change a 2D solver into a 2D axisymmetric solver:
  - r factor
  - Axis BC
- Solvers verified with manufactured and analytical solutions
- Simulation of a subsonic high-Reynolds air flow in a torch geometry

Perspectives

- Implementation of a stabilization method
- Improvement of the axisymmetric modeling of the inlets

Thanks and bibliography

Thanks

Todd A. Oliver, Karl W. Schulz, Marc Bolinches (UT Austin)

Bibliography

- V. A. Dobrev, T. E. Ellis, T. V. Kolev and R. N. Rieben, *High-order curvilinear finite elements for axisymmetric Lagrangian hydrodynamics*, Computers & Fluids 83, pp. 58-69, 2013
- J.-L. Guermond, R. Laguerre, J. Léorat and C. Nore, *Nonlinear magnetohydrodynamics in axisymmetric heterogeneous domains using a Fourier/finite element technique and an interior penalty method*, Journal of Computational Physics 228, pp. 2739–2757, 2009
- A. Ern and J.-L. Guermond, *Theory and Practice of Finite Elements*, 1st ed., Springer, New York, 2004

Thank you for your attention

Air flow in a torch geometry: Simulation (u_r)