Laser Plasma Modeling with High-Order Finite Elements

MFEM Community Workshop 2021

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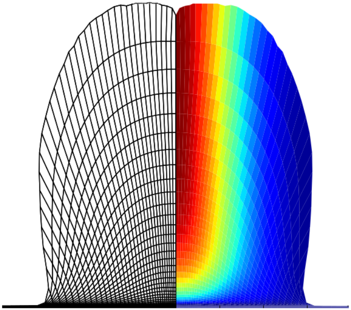
Outline

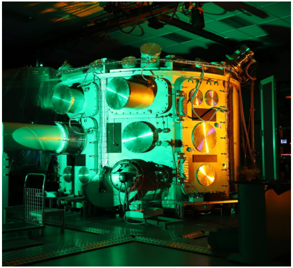


Introduction

Laser plasma modeling

- laser – target interaction, absorption, refraction, scattering, . . .
- plasma – ablation, WDM, energy transport, mag. fields, . . .
- modeling – curvilinear, DG, positive, mixed, hybridized, . . .

L4 laser system at ELI Beamlines
(10 PW, 1.5 kJ, 150 fs, 1057 nm)



P3 vacuum chamber
(5 lasers, ⌀ 5 m, 45 m^3)

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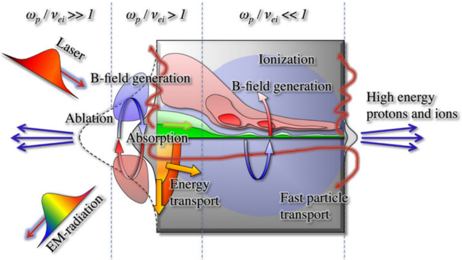
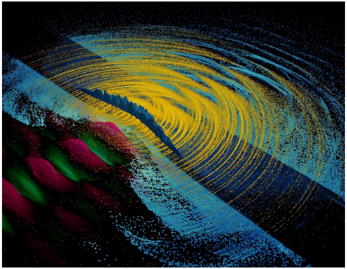


The physics of laser–target interaction
Thomas, A. G. R. et al. JCP, 231, 1051-1079 (2012)



X-ray photons production by laser–target interaction

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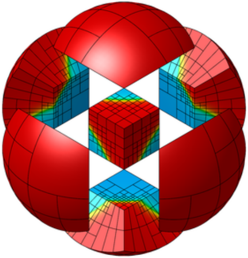
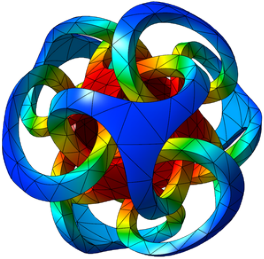


MFEM Finite Element Discretization Library



GLVis OpenGL Finite Element Visualization Tool

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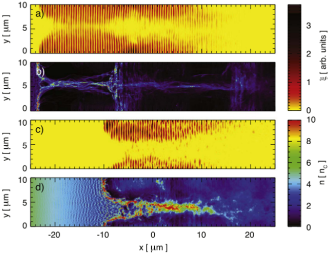
Applications

- prepulses of ultra-intense lasers
  (electron-positron pairs, vacuum Cherenkov radiation, Hawking radiation, gamma flashes, . . .)
- ion acceleration beamlines
  (hadrontherapy, proton radiography, nuclear physics, material science, . . .)
- laboratory astrophysics
- inertial confinement fusion
- . . .



Pulse filamentation in preplasma
Holec, M., Nikl, J., Vranic, M., & Weber, S. PPCF, 60(4), 044019 (2018)

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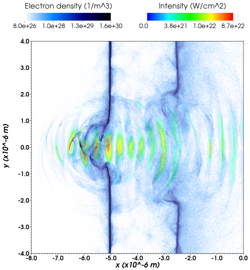

Double foil contrast enhancement
Nikl, J., Jirka, M., Matys, M., Kuchařík, M., & Klimo, O.
Proc. of SPIE, 11777, 117770X (2021)

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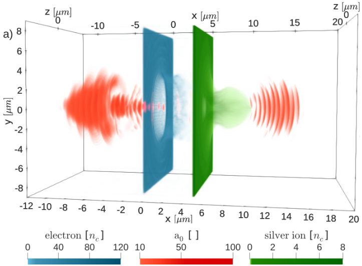

Laser ion accel. w/ plasma shutter
Matys, M. et al. Proc. of SPIE, 11779, 117790Q (2021)

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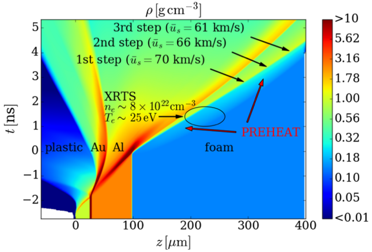
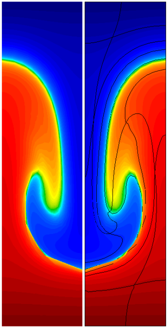


Shock preheats at Omega
Falk, K. et al. PRL, 120, 025002 (2018)

Two-temperature hydrodynamics I

- inviscid compressible quasi-neutral fluid
- Lagrangian formulation – curvilinear FE
- mass, momentum and energy – density (ρ), velocity ($\vec{u}$), temperatures (T_e, T_i)



$$\frac{\partial \rho}{\partial t} = -\rho \nabla \cdot \vec{u}$$

$$\rho \frac{\partial \vec{u}}{\partial t} = -\nabla (p_i + p_e)$$

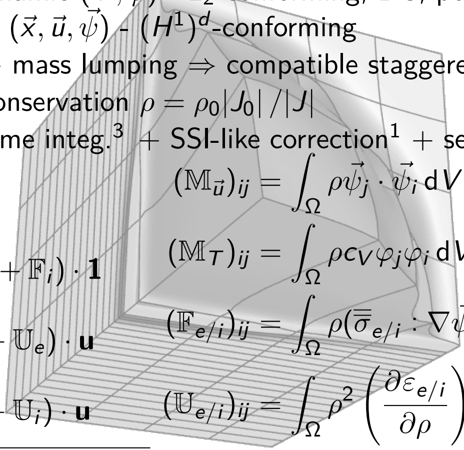
$$\rho \left(\frac{\partial \varepsilon_e}{\partial T_e} \right)_\rho \frac{\partial T_e}{\partial t} = -p_e \nabla \cdot \vec{u} + \rho^2 \left(\frac{\partial \varepsilon_e}{\partial \rho} \right)_{T_e} \nabla \cdot \vec{u} + G_{ei}(T_i - T_e)$$

$$\rho \left(\frac{\partial \varepsilon_i}{\partial T_i} \right)_\rho \frac{\partial T_i}{\partial t} = -p_i \nabla \cdot \vec{u} + \rho^2 \left(\frac{\partial \varepsilon_i}{\partial \rho} \right)_{T_i} \nabla \cdot \vec{u} + G_{ie}(T_e - T_i)$$

$+$EOS$(p_e, p_i, \varepsilon_e, \varepsilon_i)$ + collision frequency(G_{ei}, G_{ie})

Two-temperature hydrodynamics II

- high-order curvilinear finite element hydrodynamics[1,2]
  - thermodynamic (T, φ) - L_2-conforming, DG, positive
  - kinematic $(\vec{x}, \vec{u}, \vec{\psi})$ - $(H^1)^d$-conforming
- lowest order + mass lumping ⇒ compatible staggered hydrodynamics
- strong mass conservation $\rho = \rho_0 |J_0| / |J|$
- conservative time integ.[3] + SSI-like correction[1] + semi-analytic relax.



$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{u}$$

$$\mathbb{M}_{\vec{u}} \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -(\mathbb{F}_e + \mathbb{F}_i) \cdot \mathbf{1}$$

$$\mathbb{M}_{T_e} \frac{\mathrm{d}\mathbf{T}_e}{\mathrm{d}t} = (\mathbb{F}_e^T + \mathbb{U}_e) \cdot \mathbf{u}$$

$$\mathbb{M}_{T_i} \frac{\mathrm{d}\mathbf{T}_i}{\mathrm{d}t} = (\mathbb{F}_i^T + \mathbb{U}_i) \cdot \mathbf{u}$$

$$(\mathbb{M}_{\vec{u}})_{ij} = \int_\Omega \rho \vec{\psi}_j \cdot \vec{\psi}_i \, \mathrm{d}V$$

$$(\mathbb{M}_T)_{ij} = \int_\Omega \rho c_V \varphi_j \varphi_i \, \mathrm{d}V$$

$$(\mathbb{F}_{e/i})_{ij} = \int_\Omega \rho(\overline{\overline{\sigma}}_{e/i} : \nabla \vec{\psi}_j) \varphi_i \, \mathrm{d}V$$

$$(\mathbb{U}_{e/i})_{ij} = \int_\Omega \rho^2 \left(\frac{\partial \varepsilon_{e/i}}{\partial \rho} \right)_{T_{e/i}} \nabla \cdot \vec{\psi}_j \varphi_i \, \mathrm{d}V$$

[1] Nikl, J., Kuchařík, M., Holec, M., & Weber, S. Europhysics Conference Abstracts, 42A, P1.2019 (2018).

[2] Dobrev, V., Kolev, T., & Rieben, R. SIAM, 34(5), B606-B641 (2012).

[3] Sandu, A., Tomov, V., Cervena, L., & Kolev, T. SIAM, 43(1), A221–A241 (2021).

Laser absorption / X-ray amplification

- WKB absorption
  $(\vec{n}\cdot\nabla)I_l = -\alpha I_l, \quad \alpha = 2k_0 \mathrm{Im}\,\hat{n}$
  ($\hat{n} = \sqrt{\hat{\varepsilon}}$ – complex refr. index)
  upwinded DG FEs
- ray-tracing[1]
  $$\frac{\mathrm{d}}{\mathrm{d}s}\left(n\frac{\mathrm{d}\vec{r}}{\mathrm{d}s}\right) = \nabla n$$
  inv. Bremsstrahlung + resonant abs. + Fresnel + X-ray ampl.
  high-order $\leftrightarrow$ low-order-refined
- wave-based absorption[2]
  $H' + ik_0\hat{\varepsilon}E = 0, \quad E' + ik_0 H = 0$
  1D model – semi-anal + FEM
  (rasteriazation for multi-D)

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[1] Šach, M. Hydrodynamic simulations of X-ray generation and propagation in laser-produced plasmas. FNSPE CTU, 2021.
[2] Nikl, J., Kuchařík, M., Limpouch, J., Liska, R., & Weber, S. Adv Comput Math, 45(4), 1953–1976 (2019).

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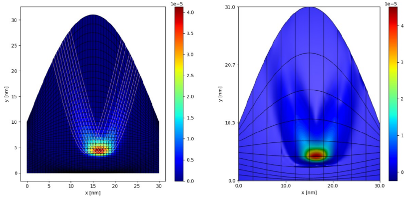

Ray-tracing absorption LOR→HO

[1] Šach, M. Hydrodynamic simulations of X-ray generation and propagation in laser-produced plasmas. FNSPE CTU, 2021.
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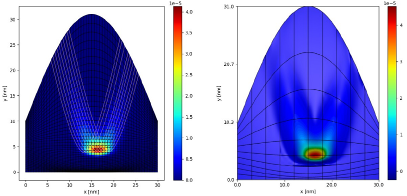
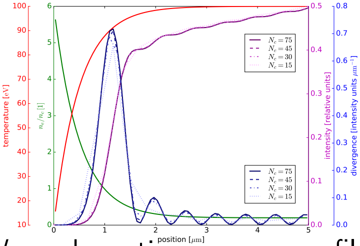


Ray-tracing absorption LOR→HO

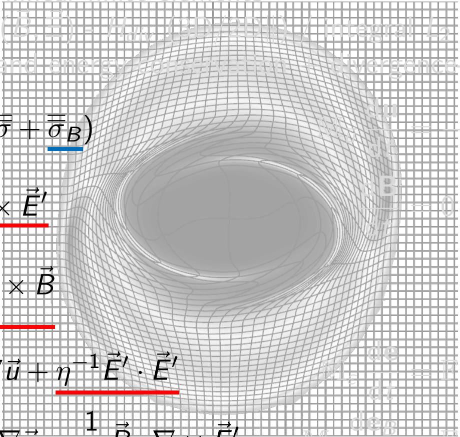


Wave absorption on exp. profiles

[1] Šach, M. Hydrodynamic simulations of X-ray generation and propagation in laser-produced plasmas. FNSPE CTU, 2021.
[2] Nikl, J., Kuchařík, M., Limpouch, J., Liska, R., & Weber, S. Adv Comput Math, 45(4), 1953–1976 (2019).

Resistive magneto-hydrodynamics I

- ideal + resistive MHD
- high-order curvilinear finite elements[1]
  - magnetic $(\vec{B}, \vec{E})$ - H_{div} (3D/2D∥) / integral L_2 (2D⊥/1D)
- magnetic flux and energy conservation → divergence-free mag. field



$$\rho \frac{\mathrm{d}\vec{u}}{\mathrm{d}t} = \nabla \cdot (\bar{\bar{\sigma}} + \bar{\bar{\sigma}}_B)$$

$$\frac{\mathrm{d}\vec{B}}{\mathrm{d}t} = -\nabla \times \vec{E}'$$

$$\frac{1}{\eta}\vec{E}' = \frac{1}{\mu_0}\nabla \times \vec{B}$$

$$\rho \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \bar{\bar{\sigma}} : \nabla\vec{u} + \eta^{-1}\vec{E}' \cdot \vec{E}'$$

$$\rho \frac{\mathrm{d}\varepsilon_B}{\mathrm{d}t} = \bar{\bar{\sigma}}_B : \nabla\vec{u} - \frac{1}{\mu_0}\vec{B} \cdot \nabla \times \vec{E}'$$

$$\mathbb{M}_V \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -(\mathbb{F} + \mathbb{F}_B) \cdot \mathbf{1}$$

$$\mathbb{M}_\varepsilon \frac{\mathrm{d}\mathbf{e}}{\mathrm{d}t} = \mathbb{F}^T \cdot \mathbf{u} + \mathbf{e}_B^c$$

$$\mathbb{M}_\varepsilon \frac{\mathrm{d}\mathbf{e}_B}{\mathrm{d}t} = \mathbb{F}_B^T \cdot \mathbf{u}$$

[1] Nikl, J., Kuchařík, M., & Weber, S. JCP, Submitted (2021).

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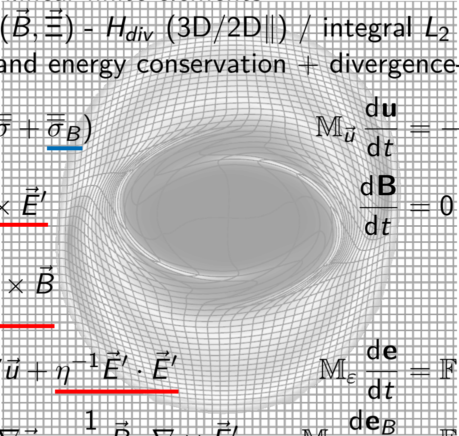

$$\rho \frac{d\vec{u}}{dt} = \nabla \cdot (\overline{\overline{\sigma}} + \overline{\overline{\sigma}}_B) \qquad \mathbb{M}_{\vec{u}} \frac{d\mathbf{u}}{dt} = -(\mathbb{F} + \mathbb{F}_B) \cdot \mathbf{1}$$

$$\frac{d\vec{B}}{dt} = -\nabla \times \vec{E}' \qquad \frac{d\mathbf{B}}{dt} = 0$$

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$$\rho \frac{d\varepsilon}{dt} = \overline{\overline{\sigma}} : \nabla\vec{u} + \eta^{-1}\vec{E}' \cdot \vec{E}' \qquad \mathbb{M}_\varepsilon \frac{d\mathbf{e}}{dt} = \mathbb{F}^T \cdot \mathbf{u} + \mathbf{e}_B^c$$

$$\rho \frac{d\varepsilon_B}{dt} = \overline{\overline{\sigma}}_B : \nabla\vec{u} - \frac{1}{\mu_0}\vec{B} \cdot \nabla \times \vec{E}' \qquad \mathbb{M}_\varepsilon \frac{d\mathbf{e}_B}{dt} = \mathbb{F}_B^T \cdot \mathbf{u}$$

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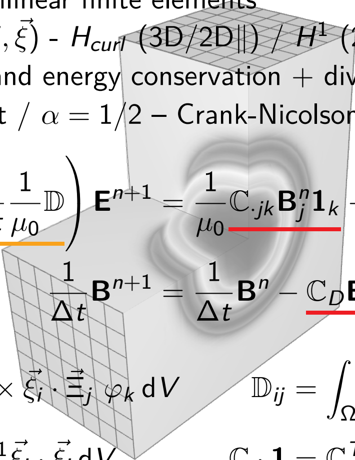
Resistive magneto-hydrodynamics II

- high-order curvilinear finite elements[1]
  - electric $(\vec{E}, \vec{\xi})$ - H_{curl} (3D/2D∥) / H^1 (2D⊥/1D)
- magnetic flux and energy conservation + divergence-free mag. field
- $\alpha = 0$ – explicit / $\alpha = 1/2$ – Crank-Nicolson / $\alpha = 1$ – fully implicit



$$\left(\mathbb{M}_{\vec{E}} + \frac{\alpha}{\Delta t}\frac{1}{\mu_0}\mathbb{D}\right)\mathbf{E}^{n+1} = \frac{1}{\mu_0}\mathbb{C}_{\cdot jk}\mathbf{B}_j^n\mathbf{1}_k - \frac{(1-\alpha)}{\Delta t}\frac{1}{\mu_0}\mathbb{D}\mathbf{E}^n$$

$$\frac{1}{\Delta t}\mathbf{B}^{n+1} = \frac{1}{\Delta t}\mathbf{B}^n - \mathbb{C}_D\mathbf{E}^{n+\alpha}$$

$$\mathbb{C}_{ijk} = \int_\Omega \nabla \times \vec{\xi}_i \cdot \vec{\Xi}_j\, \varphi_k \,\mathrm{d}V \qquad \mathbb{D}_{ij} = \int_\Omega \nabla \times \vec{\xi}_j \cdot \nabla \times \vec{\xi}_i \,\mathrm{d}V$$

$$(\mathbb{M}_{\vec{E}})_{ij} = \int_\Omega \eta^{-1}\vec{\xi}_j \cdot \vec{\xi}_i \,\mathrm{d}V \qquad \mathbb{C}\cdot\mathbf{1} = \mathbb{C}_D^T\mathbb{M}_{\vec{B}},\ \mathbb{D} = \mathbb{C}_D^T\mathbb{M}_{\vec{B}}\mathbb{C}_D$$

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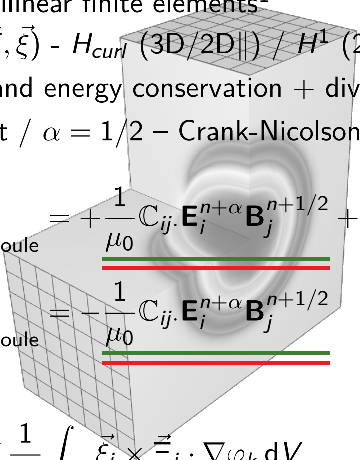

$$\mathbb{M}_T \left.\frac{\mathrm{d}\mathbf{e}}{\mathrm{d}t}\right|_{\text{Joule}} = +\frac{1}{\mu_0}\mathbb{C}_{ij.}\mathbf{E}_i^{n+\alpha}\mathbf{B}_j^{n+1/2} + \mathbb{S}_{ij.}\mathbf{E}_i^{n+\alpha}\mathbf{B}_j^{n+1/2}$$

$$\mathbb{M}_T \left.\frac{\mathrm{d}\mathbf{e}_B}{\mathrm{d}t}\right|_{\text{Joule}} = -\frac{1}{\mu_0}\mathbb{C}_{ij.}\mathbf{E}_i^{n+\alpha}\mathbf{B}_j^{n+1/2}$$

$$\mathbb{S}_{ijk} = \sum_e \frac{1}{\mu_0}\int_{\Omega_e} \vec{\xi}_i \times \vec{\Xi}_j \cdot \nabla\varphi_k \,\mathrm{d}V \qquad \mathbb{S}\cdot\mathbf{1} = \mathbb{O}$$

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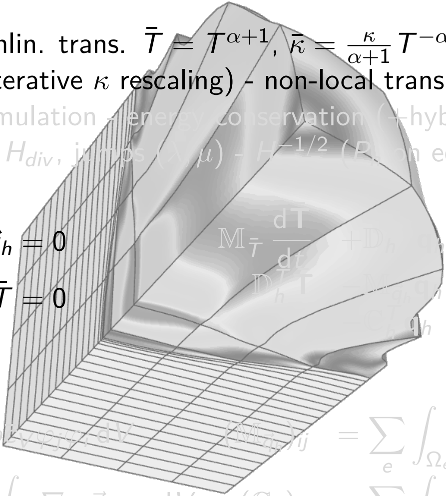
Flux-limited heat diffusion

- $\kappa \sim T^{\alpha} \Rightarrow$ nonlin. trans. $\bar{T} = T^{\alpha+1}$, $\bar{\kappa} = \frac{\kappa}{\alpha+1} T^{-\alpha}$, $\bar{c}_V = \frac{c_V}{\alpha+1} T^{-\alpha}$
- flux limiters (iterative κ rescaling) - non-local transport?
- dual (flux) formulation - energy conservation (+hybridization)
- fluxes $(\vec{q}, \vec{w})$ - H_{div}, jumps (λ, μ) - $H^{-1/2}$ (P_i on edges for RT)



$$\rho \bar{c}_V \frac{\mathrm{d}\bar{T}}{\mathrm{d}t} + \nabla \cdot \vec{q}_h = 0$$
$$\vec{q}_h + \bar{\kappa} \nabla \bar{T} = 0$$

$$\mathbb{M}_{\bar{T}} \frac{\mathrm{d}\mathbf{T}}{\mathrm{d}t} + \mathbb{D}_h \mathbf{q}_h = 0$$
$$\mathbb{D}_h^T \mathbf{T} - \mathbb{M}_{q_h} \mathbf{q}_h - \mathbb{C}_h \lambda = 0$$
$$\mathbb{C}_h^T \mathbf{q}_h = 0$$

$$(\mathbb{M}_{\bar{T}})_{ij} = \int_{\Omega} \rho \bar{c}_V \varphi_j \varphi_i \, \mathrm{d}V \qquad (\mathbb{M}_{q_h})_{ij} = \sum_e \int_{\Omega_e} \bar{\kappa}^{-1} \vec{w}_j \cdot \vec{w}_i \, \mathrm{d}V$$

$$(\mathbb{D}_h)_{ij} = \sum_e \int_{\Omega_e} \nabla \cdot \vec{w}_j \, \varphi_i \, \mathrm{d}V \qquad (\mathbb{C}_h)_{ij} = \sum_e \oint_{\partial\Omega_e} \mu_j \, \vec{w}_i \cdot \mathrm{d}\vec{S}$$

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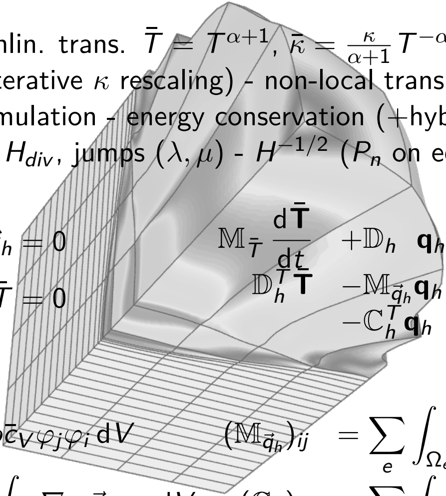

$$\rho \bar{c}_V \frac{\mathrm{d}\bar{T}}{\mathrm{d}t} + \nabla \cdot \vec{q}_h = 0 \qquad \vec{q}_h + \bar{\kappa} \nabla \bar{T} = 0$$

$$\begin{aligned} \mathbb{M}_{\bar{T}} \frac{\mathrm{d}\bar{\mathbf{T}}}{\mathrm{d}t} & + \mathbb{D}_h \ \mathbf{q}_h & = 0 \\ \mathbb{D}_h^T \bar{\mathbf{T}} & - \mathbb{M}_{\vec{q}_h} \mathbf{q}_h - \mathbb{C}_h \lambda & = 0 \\ & - \mathbb{C}_h^T \mathbf{q}_h & = 0 \end{aligned}$$

$$(\mathbb{M}_{\bar{T}})_{ij} = \int_\Omega \rho \bar{c}_V \varphi_j \varphi_i \,\mathrm{d}V \qquad (\mathbb{M}_{\vec{q}_h})_{ij} = \sum_e \int_{\Omega_e} \bar{\kappa}^{-1} \vec{w}_j \cdot \vec{w}_i \,\mathrm{d}V$$

$$(\mathbb{D}_h)_{ij} = \sum_e \int_{\Omega_e} \nabla \cdot \vec{w}_j \ \varphi_i \,\mathrm{d}V \qquad (\mathbb{C}_h)_{ij} = \sum_e \oint_{\partial\Omega_e} \mu_j \ \vec{w}_i \cdot \mathrm{d}\vec{S}$$

Non-local energy transport

- **BGK model**[1]: first-principle model, empirical col. operator, arb. anisotropy, covers diffusion limit, local electric field

$$\frac{\partial f_e}{\partial t} + \vec{v}_e \cdot \nabla_{\vec{x}} f_e + \frac{q_e}{m_e} E \cdot \nabla_{\vec{v}} f_e = \frac{n_i}{n_e} \frac{\mathrm{d}\bar{Z}}{\mathrm{d}t} f_S + \nu_{ei}(f_S - f_e) + \nu_{\sigma_e}(\bar{f}_e - f_e)$$

- intensity $I_e = \int \frac{1}{2} m_e |\vec{v}|^5 f_e \,\mathrm{d}|\vec{v}|, p \in \{e, R\}, c_R = c, c_e = +\infty$

$$\rho c_{V e} \frac{\mathrm{d}T_e}{\mathrm{d}t} + \int_{4\pi} \frac{1}{c_p} \frac{\mathrm{d}I_p}{\mathrm{d}t} + \vec{n} \cdot \nabla I_p \,\mathrm{d}\omega = 0$$

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- linearisation of the source[2] $\Rightarrow$ implicit coupling $S_p = S_A^p T_e + S_b^p$
- hybridized DG in space + DG/H^1 in angles ($\sim$ discrete ordinates)

[1] Holec, M., Nikl, J., & Weber, S. PoP, 25(3), 032704 (2018).

[2] Holec, M., Limpouch, J., Liska, R., & Weber, S. Int. J. Numer. Meth. Fl., 83(10), 779–797 (2017).

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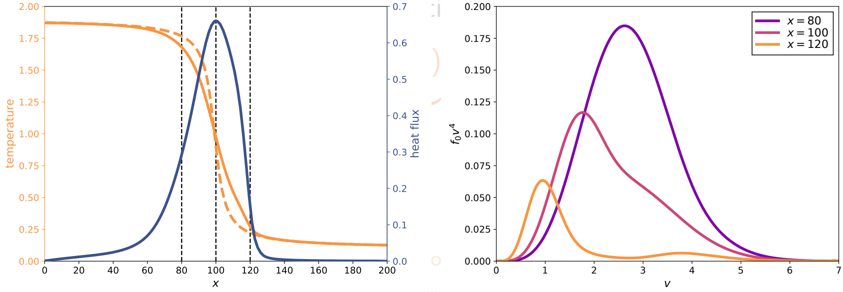
Vlasov–Fokker–Planck–Maxwell

- transient spectrally-resolved non-local transport + EM fields



Heat flux over a steep slope of temperature (left) and the f_0 distribution function at the marked points (right)

- high-order FEM in space ($f_0 \in L_2(\Omega)$, $\vec{f}_1 \in (H^1)^3(\Omega)$, $\vec{E} \in H_{div}(\Omega)$, $\vec{B} \in H_{curl}(\Omega)$) + staggered FD in velocities[1]
- mass, charge, energy conservation + fully implicit

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- Cartesian tensor expansion $f_e(\vec{x}, v, \vec{n}, t) = f_0(\vec{x}, v, t) + \vec{f}_1(\vec{x}, v, t) \cdot \vec{n}$
- diffusion limit $\Rightarrow$ Braginskii resistive MHD

$$\frac{\partial f_0}{\partial t} + \underbrace{\frac{v}{3}\nabla \cdot \vec{f}_1}_{\text{flux. div.}} \underbrace{- \frac{e}{m_e}\frac{1}{3v^2}\frac{\partial}{\partial v}\left(v^2 \vec{E} \cdot \vec{f}_1\right)}_{\text{Joule heating}} = \frac{\bar{\nu}_{ee}}{v^2}\frac{\partial}{\partial v}\left(\underbrace{C(f_0) f_0}_{\text{dyn. friction}} + \underbrace{D(f_0)\frac{\partial f_0}{\partial v}}_{\text{diffusion}} \right)$$

$$\frac{\partial \vec{f}_1}{\partial t} \underbrace{+ v \nabla f_0}_{\text{pot. grad.}} \underbrace{- \frac{e}{m_e}\vec{E}\frac{\partial f_0}{\partial v}}_{\text{el. field acceleration}} \underbrace{- \frac{e}{m_e}\vec{B} \times \vec{f}_1}_{\text{Hall term}} = \underbrace{-\nu_s \vec{f}_1}_{\text{scaterring}}$$

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Conclusions

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  - laser absorption/generation
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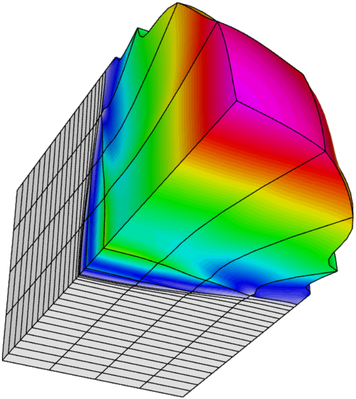
⇒ hybrid modeling, non-local electron transport, extended MHD?

⇒ **a postdoc position?** (from Oct 2022)



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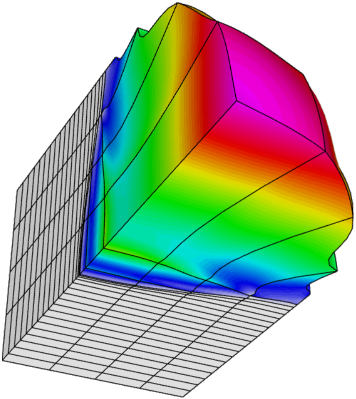

Thank you for your attention