Scalable Design and Optimization with MFEM

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Boyan Lazarov

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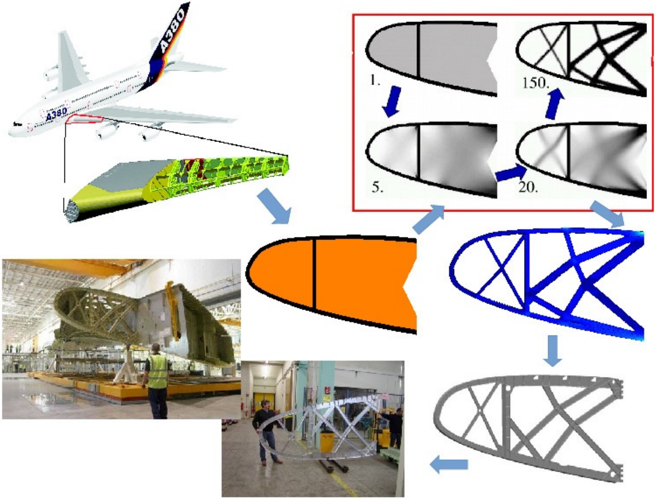
Minimize weight/maximize stiffness

Airbus A380

Topology optimization in aerospace



Topology optimization

Density based topology optimization

$$
\begin{aligned}
\min_{\rho} \; :& g_0\left(\widetilde{\rho}, u\right) \\
s.t. \; :& r_p\left(\widetilde{\rho}, u\right) = 0, \quad u \in \mathcal{U}_{\text{ad}} \\
& r_f\left(\widetilde{\rho}, \rho\right) = 0, \quad \widetilde{\rho} \in \widetilde{\mathcal{D}}_{\text{ad}} \\
& \cdots\cdots \\
& g_i\left(\widetilde{\rho}, u\right) \leq 0, \quad i = 1 \ldots N_g \\
& \rho \in \mathcal{D}_{\text{ad}}
\end{aligned}
$$

Topology optimization - general algorithm

Set $\rho \leftarrow \rho_0$

repeat

Solve filter PDE: $-\nabla^{\mathsf{T}} r^2 \nabla \widetilde{\rho} + \widetilde{\rho} = \rho, \; \rightarrow \widetilde{\rho}\,(\rho)$

Solve state PDE: $-\nabla^{\mathsf{T}} \kappa\,(\widetilde{\rho})\, \nabla u = f, \; \rightarrow u\,(\widetilde{\rho}\,(\rho))$

Evaluate QoI: $g_i\,(u\,(\widetilde{\rho}\,(\rho)))\,, i = 0, 1, \ldots, N_g \; \rightarrow g_i$

Solve adjoints:

$-\nabla^{\mathsf{T}} \kappa^{\mathsf{T}}\,(\widetilde{\rho})\, \nabla \lambda_i = -g_i'\,(u)\,, \; i = 0, 1, \ldots, N_g \quad \rightarrow \lambda_i$

Evaluate gradients: $g'_{i,\widetilde{\rho}} = \frac{\partial \Lambda\left(u, \lambda; \widetilde{\rho}\right)}{\partial \widetilde{\rho}} \; \rightarrow g'_{i,\widetilde{\rho}}$

Apply chain rule: $g'_{i,\widetilde{\rho}} \rightarrow g'_{i,\rho}$

Update ρ - popular algorithms OC, MMA, Ipopt, HiOp, SQP;

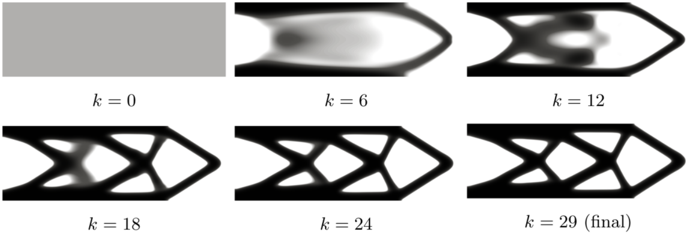
until *a convergence test is satisfied*

MFEM ingredients for topology optimization

- Grid functions - represent state and design fields on a FE mesh
- Scalable PDE discretizations - bilinear, linear, and non-linear forms for FE discretization
- Scalable algebraic solvers - solve $\mathbf{Ku} = \mathbf{f}$
- Scalable preconditioners - algebraic and geometric multigrid
- Powerful abstractions for coefficients

Example 37: Topology Optimization [Keith and Surowiec, 2023]

- Isotropic linear elastic problem
- SIMP interpolation
- Design update - Entropic mirror descent algorithm



Topology Optimization with uncertain excitation [Bollapragada et al., 2023]

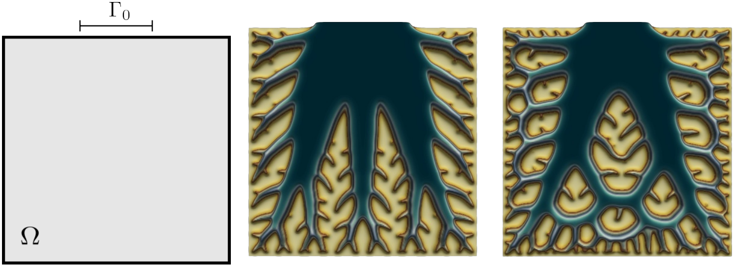
The full problem formulation is written as follows:

$$\min_{\rho \in L^2(\Omega),\, u \in H^1(\Omega)} \left\{ \widehat{F}(\rho, u) := \mathbb{E}\left[\int_\Omega u f \, \mathrm{d}\mathbf{x} \right] \right\},$$

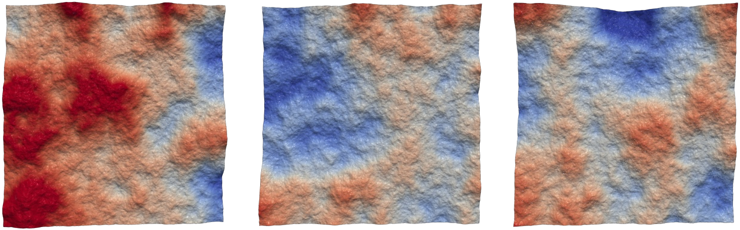
subject to the constraints

$$\begin{cases} -\epsilon^2 \Delta \widetilde{\rho} + \widetilde{\rho} = \rho \ \text{ in } \Omega\,, \quad \nabla \widetilde{\rho} \cdot \mathbf{n} = 0 \ \text{ on } \partial\Omega\,, \\ -\mathrm{div}\big(r(\widetilde{\rho}) \nabla u\big) = f \ \text{ in } \Omega\,, \quad u = 0 \ \text{ on } \Gamma_0\,, \quad \nabla u \cdot \mathbf{n} = 0 \text{ on } \partial\Omega \setminus \Gamma_0\,, \\ \int_\Omega \rho(\mathbf{x}) d\mathbf{x} \leq \gamma |\Omega|\,, \quad \text{and} \quad 0 \leq \rho \leq 1 \ \text{ in } \Omega\,, \end{cases}$$

Topology Optimization with uncertain excitation [Bollapragada et al., 2023]

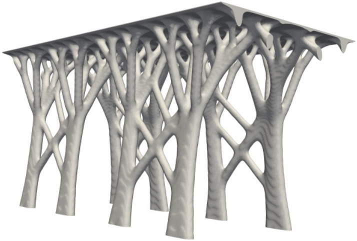


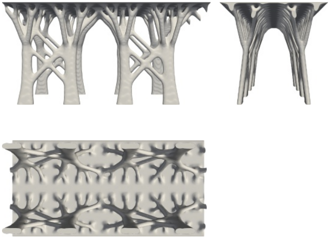
Random heat influx



Topology Optimization with uncertain excitation

Bridge design

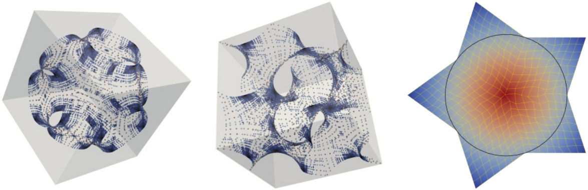




Heat sinks

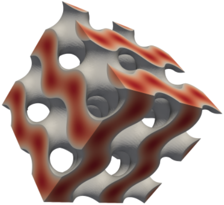
Body-fitted integration on implicit geometries

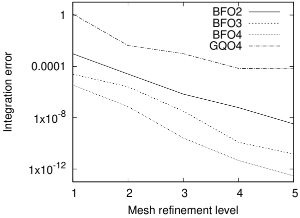
- Does not require any geometric operations.
- Evaluation of surface integrals on grid functions.
- Evaluation of volume integrals to an arbitrary precision.
- Implemented for 2D and 3D quad and hex meshes.
- Implementation for triangles and tetrahedrons in progress.



Body-fitted integration on implicit geometries

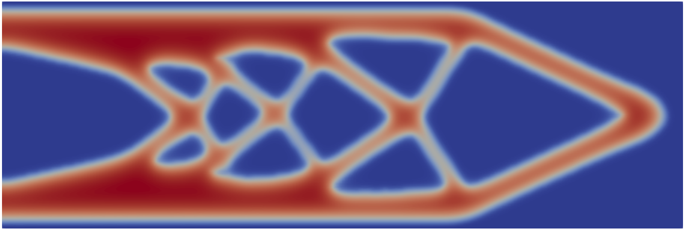
- Volume integration of implicitly defined Gyroid topology with Heaviside cut-off
  - body-fitted with MFEM-ALGOIM interface
  - standard Gaussian integration



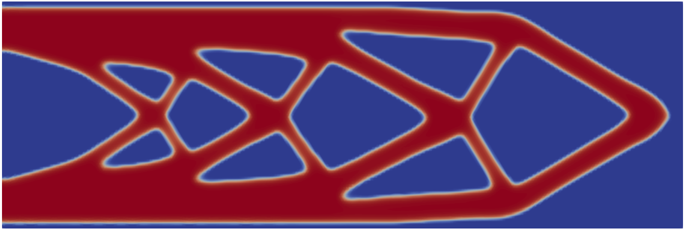


Density based topology optimization [Wang et al., 2011, Lazarov et al., 2016]

Standard SIMP penalization (two fields) $\rho \to \widetilde{\rho}$

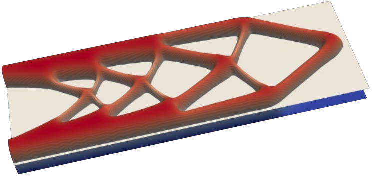


Projected SIMP penalization (three fields) $\rho \to \widetilde{\rho} \to \hat{\rho}$

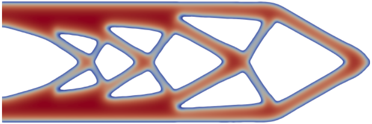


Shape optimization as postprocessing step

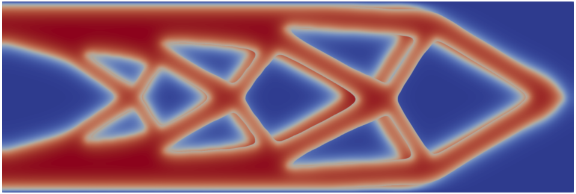
LS function = density



Shape optimized topology

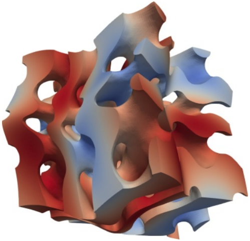


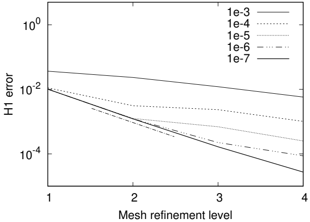
Comparison



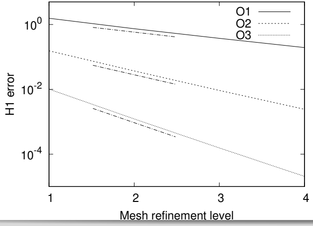
High-order CutFEM solvers for shape optimization

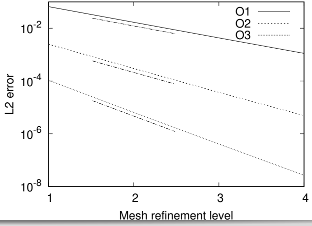
Deformed Gyroid structure stabilized with weak material.





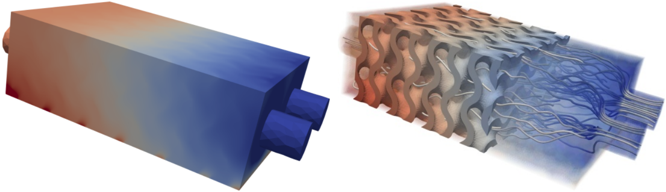
Stabilized CutFEM solution for linear, quadratic, and cubic elements.



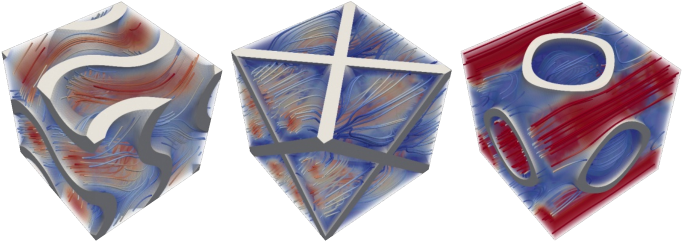


Homogenization

Full scale solution

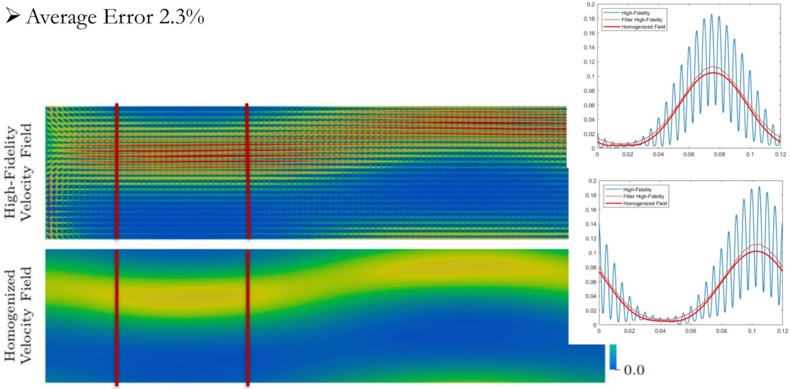


Unit cell solutions - ML interpolation $\mathbf{v} = f\left(\nabla p\right)$



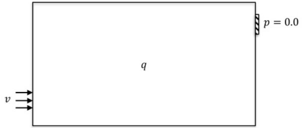
Homogenization

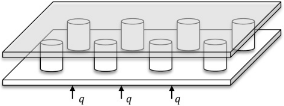
- Verification of homogenized analysis against high fidelity model
- Average Error 2.3%

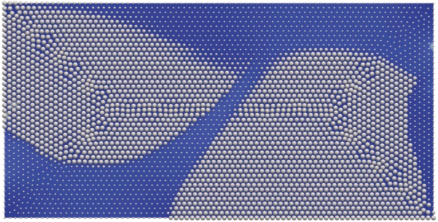


Homogenization - optimization of heat exchanger

Heat Exchanger:







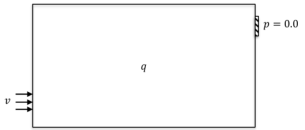
Optimization Formulation:

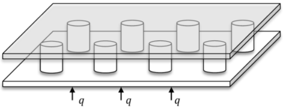
$$\min_{s} \quad z(s, p(s)) = T^T K T$$

$$\text{s.t.} \quad g_i(s) \leq 0, \ i = 1, \dots, N_g$$

Homogenization - optimization of heat exchanger

Heat Exchanger:





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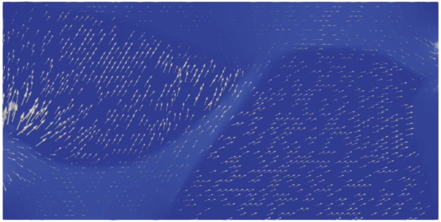

New features for design and optimization

- Automatic differentiation:
  - Native MFEM - forward mode.
  - CoDiPack - reverse and forward mode.
  - Enzyme - reverse and forward.
- CPU and GPU ready native MFEM MMA optimizer.
- Native MFEM integration for immersed discretizations.
- Immersed solvers and adjoints for linear/nonlinear elasticity, diffusion, advection-diffusion, Stokes.

References I

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Thank you for your attention. Any questions?

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